CHAPTER 36

MATHEMATICS

Doctoral Theses

01. AMIT KUMAR
ρ - Operator Radius Inequalities for Hilbert Space Operators.
Supervisor: Prof. Shiv Kumar Kaushik
Th 29153

Abstract

The thesis entitled "ρ-Operator Radius Inequalities for Hilbert Space Operators" is devoted to the study of ρ-operator radius inequalities for operators on Hilbert spaces, with particular emphasis on numerical radius inequalities. Some new generalizations and refinements of numerical radius inequalities are obtained for finite-dimensional operators, including bounds involving higher powers of operators, anti-commutators, Hadamard products, and extensions to normalized unitarily invariant norms. Us- ing techniques from dilation theory, sharp two-sided bounds for $\omega_\varrho(A)$ in terms of $\|AA^* + A^*A\|$ are established. The results are further extended to analytic functional inequalities. In addition, a characterization of normal ρ-operator contractions on finite-dimensional Hilbert spaces is obtained. The multiplicative behavior of the ρ-operator radius for commuting bounded operators is also examined. In particular, submultiplicativity of $\omega_\varrho(\cdot)$ is established for $0 < \rho \leq 1$, and refined upper bounds for the ρ-operator radius of operator products are obtained for $0 < \rho \leq 2$, improving earlier results related to the Holbrook conjecture. Moreover, the behavior of the sequence $\{\|A^n g\|\}_{n\geq 0}$, where $A \in \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{H}})$ is a numerical-radius contraction and g is a unit vector, is investigated. A key result establishes the connection between the sequence $\{\|A^n g\|\}_{n\geq 0}$ and chain sequences. As application of this result, an alternate proof of Crabb`s theorem is given, analyzed the monotone, constant, and the geometric cases of the sequence $\{\|A^n g\|\}_{n\geq 0}$, and provide refinement of some results due to Ando and Berger. Finally, using Ando's characterization of numerical-radius contractions, a structure theorem is obtained that explains the shift operator like behaviour of operator A for a broad class of extremal sequences $\{\|A^n g\|\}_{n\geq 0}$.

Contents

1. Introduction 2. Numerical Radius Inequalities via Maximization 3. Generalized Operator Radius Inequalities 4. Operator Radii of Commuting Products 5. Norm Iterative Sequences of Numerical Radius Contractions and Chain Sequences. References.

02. APARNA
Mathematical Formulation and Solution Techniques for Fuzzy Fixed Charge and Neutrosophic Transshipment Problems.
Supervisor: Prof. Ratnesh Rajan Saxena
Th 29154

Abstract

The transshipment problem is an extension of the classical transportation problem in which goods are allowed to move through intermediate nodes before reaching their final destinations. This structure enables bidirectional flow between sources and destinations and provides a flexible and realistic framework for modeling complex logistics and distribution networks. This thesis focuses on the formulation and solution of multiple classes of transshipment problems under uncertain environments, where uncertainty is modeled using fuzzy and neutrosophic set frameworks. The work focuses on fuzzy transshipment problems, fixed-charge fuzzy transshipment problems, and neutrosophic transshipment problems modeled through different forms of pentagonal fuzzy and neutrosophic numbers. In fuzzy transshipment problems, uncertainty and imprecision in parameters such as cost, supply, and demand are represented using asymmetric pentagonal fuzzy numbers. A ranking function technique is employed to transform fuzzy parameters into comparable crisp values. Using this ranking mechanism, a computationally efficient solution technique is developed for balanced fuzzy transshipment problems, yielding a direct fuzzy optimal solution while significantly reducing computational effort. The thesis also addresses fixed-charge fuzzy transshipment problems, where an additional fuzzy fixed cost is incurred whenever a shipment is made, irrespective of the quantity transported. An approximate solution methodology is proposed by determining lower and upper bounds for the optimal objective value using asymmetric pentagonal fuzzy numbers, effectively incorporating fixed charges while maintaining solution feasibility. While fuzzy set theory models vagueness, it does not explicitly capture indeterminacy arising from incomplete or conflicting information. To overcome this limitation, the transshipment problem is formulated in a neutrosophic environment. Problem parameters are represented using single-valued pentagonal neutrosophic numbers, and a ranking function based on alpha cuts and possibility mean values is employed to obtain comparable crisp equivalents. A direct solution methodology is developed to determine the optimal solution of the neutrosophic transshipment problem.

Contents

1. Introduction 2. Fuzzy Transshipment Problem 3. Fixed Charge Fuzzy Transshipment Problem 4. Neutrosophic Transshipment Problem. Future Scope, List of Publications, References, AMS MSC 2020 Classification and Keywords.

03. DABRAL (Ajay)
Lattice-based Cryptography: Basis Reduction, Hardness Assumptions and Cryptanalysis.
Supervisors: Prof. Arvind and Dr. S. K. Pal
Th 29355

Abstract

The development of quantum algorithms has created a need for cryptographic systems expected to remain secure against quantum attacks. Among the leading approaches, lattice-based cryptography is significant because it combines practical constructions with security foundations arising from hard problems on lattices. This thesis studies lattice-based cryptography from the viewpoints of basis reduction, hardness assumptions and cryptanalysis. The thesis first examines lattice basis reduction techniques and their role in the design and analysis of lattice-based cryptographic schemes. Classical and modern methods, including LLL and BKZ, are studied with attention to their performance, limitations and improvements. A

characterisation of when a lattice vector can occur as the first vector of some lattice basis is also discussed. An experimental comparison of LLL and BKZ is carried out for bounded relation recovery in NTRU-type lattices. Under a fixed scan budget, the experiments examine how different parameter choices affect the recovery of bounded relations and identify ranges in which recovery is feasible or becomes unlikely. The thesis further studies hardness assumptions arising from lattice problems, with particular emphasis on the promised variant GapSVP with parameters ζ and γ. In this setting, a self-reduction for smaller values of ζ and γ is presented. Finally, the thesis analyses a lattice-based blind ring signature scheme proposed for anonymous blockchain transactions. It is shown that the scheme does not satisfy blindness, since a malicious signer can link a signing transcript with a valid message-signature pair. An improved blinding method is proposed, and the correctness and blindness of the modified scheme are established.

Contents

1. Introduction 2. Some Lattice Basis Reduction Techniques 3. Experimental Comparison of LLL and BKZ for Bounded Relation Recovery in NTRU-Type Lattices 4. Reductions in Lattice-Based Problems and Hardness of GapSVP 5. Security Analysis of Lattice-based Blind Ring Signature Scheme. Some Open Problems References. Appendix.

04. DEEPIKA

Complex Symmetry of Operators on Hilbert Spaces, Bergman Space and Fock Space.

Supervisor: Prof. Anuradha Gupta

Th 29155

Abstract

The thesis studies the complex symmetry of operators on Hilbert spaces, Bergman space and Fock space. We establish the necessary and sufficient conditions for the complex symmetry of H-Toeplitz operators on the Bergman space with respect to specific conjugations. Furthermore, we investigate how complex symmetry interacts with isometric, co-isometric and unitary operators. A necessary condition for the complex symmetry of slant H-Toeplitz operators on the Hardy space is also presented. We have obtained a characterization of k-normality within the class of complex symmetric operators. In addition, we prove that the complex symmetry of the Duggal transform of an invertible operator is completely determined by the complex symmetry of the original operator, thereby establishing a precise correspondence between the two. We characterize the complex symmetry of composition operators on the Hilbert space A_1^2(D). A necessary condition for the complex symmetry of weighted composition operators on this space, expressed explicitly in terms of the associated symbols is established. We have characterized conjugations on a vector-valued reproducing kernel Hilbert space. Necessary conditions are established for Toeplitz operators on the vector-valued reproducing kernel Hilbert space to exhibit complex symmetry. Also, the complex symmetry of Toeplitz operators with respect to a specific conjugation on the vector-valued reproducing kernel Hilbert space is studied. We investigate the complex symmetry of differential operators on generalized Fock spaces. We establish necessary and sufficient conditions for the boundedness of weighted composition conjugate-linear operators on the generalized Fock space. Also, we provide a complete characterization of weighted composition conjugations on the generalized Fock space, leading to a general formulation denoted by Jb,α, where b is a fixed complex parameter. Finally, we derive a necessary condition under which differential operators exhibit Jb,α-symmetry. Furthermore,

we prove that the maximal differential operator is Jb,α-symmetric if and only if it is the trivial operator.

Contents

1. Introduction 2. Complex symmetry of h-toeplitz and slant h-toeplitz operators on function spaces 3. Complex symmetries of operators on hilbert space 4. Complex symmetry of toeplitz operators on vector-valued reproducing kernel hilbert spaces 5. Complex symmetries of unbounded operator on generalized fock spaces 6. List of publications. References.

05. FAMHAWITE (Lalhmingsangi)
Numerical Simulations of Bio-Heat Transfer Process in Cryosurgical Treatment of Tumor Tissues.
Supervisors: Dr. Pooja Raj Verma and Dr. Sonam Tanwar
Th 29356

Abstract

Cryosurgery is a minimally invasive therapeutic technique that destroys abnormal biological tissues through controlled freezing. Despite its increasing clinical adoption, cryosurgery remains challenging due to the complex interaction of heat transfer, blood perfusion, phase change, and tissue response. Mathematical modeling and numerical simulation play a crucial role in understanding these interactions and improving treatment planning. Existing literatures predominantly focus on single tumor geometries, whereas multiple tumor problems introduce additional numerical complexity and interaction effects. Although blood perfusion is often modeled as a source term, explicit coupling between bioheat equations and incompressible Navier–Stokes equations is rarely addressed. The effect of blood flow in the vascular network is not sufficiently explored in the existing literatures. The resulting mathematical model exhibits strong nonlinearity due to temperature-dependent coefficients, discontinuities across phase interfaces, and coupling with fluid flow in vascular domains. Analytical solutions are intractable; hence, rigorous numerical approximation becomes essential. There is limited work treating cryoprobes positioning as cryoprobes are often randomly placed rather than mathematically optimized. This work presents a comprehensive mathematical and numerical investigation of the bioheat transfer processes involved in cryosurgical treatment of tumor tissues. The governing phenomena are modeled using nonlinear, time-dependent partial differential equations derived from the Pennes bioheat equation, suitably modified to incorporate phase change, blood perfusion, metabolic heat generation, and vascular effects. The equations are reformulated in weak form and discretized using the Galerkin Finite Element Method (FEM). Optimization problems are formulated using clustering theory and force-equilibrium systems, notably K-means clustering and bubble packing algorithms. Numerical experiments on patient specific tumor demonstrate the optimized cryoprobe configurations significantly improving the coverage of target regions under lethal temperature thresholds.

Contents

1. Introduction 2. A Study of Bio-Heat Transfer for Cryoablation of Regularly Shaped Multiple Tumors in Liver 3. A Study of Effect of Vascular Network in Liver Cryosurgery 4. Segmentation of Tumors Using Deep Learning Based Method 5. Optimization of Cryoprobes Placement in Cryosurgery. Conclusion of the Work. Future Scope of the Work, Appendixes and References.

06. MOAR (Anveksha)
Nonlinear Scalarization, Convergence of Solution Sets and Algorithms in Set Optimization.
Supervisor: Prof. C. S. Lalitha
Th 29357

Abstract

The main objective of the thesis is to present various aspects of set optimization, namely scalarization, convergence of solution sets and algorithms. We begin by studying scalarization techniques in set optimization, where the solution concept is grounded on set-order relations based on the null set. We propose two nonlinear scalarizing functions, one of the Gerstewitz type and the other based on the Hausdorff distance. To define the Gerstewitz type scalarizing function, we introduce the notion of the pseudo-algebraic interior of a nonempty subset of the hyperspace consisting of all nonempty compact convex subsets of a nontrivial real normed space. We then provide scalar characterizations of set-order relations and use it to study scalarization in set optimization. Using the Gerstewitz type scalarizing function, we explore the convergence of solution sets in set optimization. We prove the Painlevé–Kuratowski convergence of weak minimal and minimal solution sets in the domain space. Additionally, we introduce a notion of convergence in the hyperspace to analyze convergence in the image space. In the final part of the thesis, we present two algorithms based on m-order relation and lower set-order relation. The first algorithm is developed using two types of auxiliary scalar problems that can be reformulated as linear programming problems. We establish characterization of minimal and weak minimal solutions in terms of solutions of these scalar problems. These characterizations form the basis of the algorithm. In the second algorithm, we propose to apply a classical derivative-free descent method to a scalarizing function, defined in terms of the oriented distance function, to compute a weak minimal solution. Here, the Armijo–Goldstein step size rule is used to determine step length. Both the algorithms are illustrated by numerical examples.

Contents

1. Introduction 2. Nonlinear scalarization via Gerstewitz function 3. Nonlinear scalarization via Hausdorff oriented distance function 4. Painlev´e–Kuratowski convergence of solution sets 5. Algorithm in polytopic set optimization using linear optimization 6. Descent method in set optimization using oriented distance function. Bibliography.

07. NAGESHWAR (Rohit)
On Entropy, Stability and Spectral Decomposition Using Variants of Expansivity and Shadowing Property.
Supervisor: Prof. Tarun Kumar Das
Th 29358

Abstract

The theory of topological dynamics provides a rigorous mathematical framework for modeling complex, time-dependent phenomena by examining the qualitative and asymptotic behavior of systems governed by deterministic laws. This thesis work focuses on the fundamental dynamical notions of expansivity and the shadowing property, which play a central role in the study of topological stability formalized by Walters and in the spectral decomposition theorems originally established by

Smale and Bowen. Recently, researchers have introduced refined forms of these notions and explored them from both pointwise and set-valued perspectives. Motivated by above developments, we first investigate asymptotic and bi-asymptotic expansivity, introduced by Lee et al., in connection with the shadowing property, and then partially answer an open question of whether a weak bi-asymptotically expansive homeomorphism is necessarily bi-asymptotically expansive. We define asymptotically expansive points and improve results of Arbieto and Rego by proving that the existence of a non-isolated, non-wandering, non-periodic shadowable point whose neighborhood exhibits asymptotic expansivity guarantees positive topological entropy. Observing bi-asymptotically expansive behavior in c-expansive maps, we introduce bi-asymptotically c-expansive maps and examine their relationships with the two-sided s-limit shadowing property. For such systems, we establish a spectral decomposition theorem and characterize topological entropy via the growth rate of periodic orbits, thereby extending existing results. We further analyze the preservation of variants of expansivity and the shadowing property under topological semi-conjugacies induced by locally isometric covering maps, and provide criteria for their persistence under orbital and weak orbital convergence. Building upon works of Metzger et al., we introduce topologically stable points for set-valued dynamical systems and prove Walters' stability theorem in pointwise dynamics. We finally fill a potential gap in the study by establishing the equivalence of shadowing, α-persistence, and topological stability for positively expansive maps in both global and pointwise frameworks in classical and set-valued settings.

Contents

1. Introduction 2. On dynamics of asymptotically expansive maps 3. On dynamics of bi-asymptotically *c*-expansive maps 4. Dynamics under covering maps and orbital convergence 5. Topological stability of set-valued maps. Some Open Problems. Bibliography. List of Symbols and Index.

08. NITESH KUMAR

Analysis of Equilibrium Points and Attraction Zones in the Restricted Four-Body Problem with Modified Potentials.

Supervisor: Prof. Vinay Kumar

Th 29157

Abstract

The restricted three-body problem (R3BP) has long been a cornerstone of celestial mechanics, describing how a small, massless object moves under the gravity of two massive bodies orbiting their common center of mass. Building on this, the restricted planar equilateral four-body problem (R4BP) introduces a third massive body, with three primaries forming an equilateral triangle and moving in circular orbits while a fourth, infinitesimal body travels under their combined influence. Although the classical R4BP has been well studied, real astrophysical systems are far more complex, influenced by radiation pressure, asteroid belts, relativistic effects, and alternative gravitational theories. This thesis aims to bridge the gap between simplified theoretical models and the richer dynamics of real celestial environments. Using Newton–Raphson basins of convergence and basin entropy as diagnostic tools, it explores how different physical factors shape the motion and predictability of bodies within the R4BP. Chapter 1 introduces the R4BP framework, deriving the equations of motion, identifying equilibrium points, and outlining the numerical tools used throughout the work. Chapter 2 examines the effects of radiation pressure and an asteroid belt, showing how these elements increase dynamical complexity

and destroy stability. Chapter 3 studies small perturbations in Coriolis and centrifugal forces, revealing that centrifugal effects significantly alter equilibrium locations but do not lead to stable configurations. Chapter 4 incorporates relativistic corrections through a pseudo–Newtonian approach, producing intricate fractal patterns and higher unpredictability. Chapter 5 introduces Yukawa-type gravitational corrections, where the coupling constant strongly influences the system's geometry and convergence behavior. Across all models, the modifications intensify chaos, reshape equilibrium structures, and highlight the inherent instability of the R4BP. Overall, the study shows that Newton–Raphson basins and basin entropy are powerful tools for understanding the complex and often unpredictable nature of multi-body gravitational systems.

Contents

1. Introduction 2. Unveiling the attracting regions in the photogravitational four-body problem including the effect of an asteroids belt[1] 3. Investigating attraction zones in the photogravitational four-body problem: effects of asteroids belt and small perturbations in Coriolis and centrifugal forces[2] 4. Analysis of basins of attraction in the pseudo–Newtonian photogravitational restricted four–body problem with asteroid belt effects 5. Investigating the impact of Yukawa correction on the photogravitational restricted four-body problem in a circular asteroids belt.

09. PANDEY (Rakesh Kumar)
Coefficient Functionals and Certain Estimates of Analytic Functions.
Supervisors: Dr. Pratima Rai and Dr. Sushil Kumar
Th 29359

Abstract

Univalent functions play a key role in Geometric function theory that avoid self-overlapping or folding of the domain. As a result, the geometry of their image domains becomes more accessible and easier to study. In 1916, Bieberbach provided the conjecture which is related with the coefficients of the normalized univalent functions in the open unit disk. This became the central problem for many researchers of the 20th century. After remaining an open problem for over sixty years, the conjecture was finally settled in 1984 by Louis de Branges. This had stimulated extensive work on univalent functions, coefficient estimates, and related extremal questions. The geometrical interpretation of the univalent functions led to the development of the concept of starlike functions, convex functions and close-to-convex functions. The notions of subordination and functions with positive real part have played a fundamental role in the development of univalent function theory, leading to the introduction and investigation of various subclasses of univalent functions. This thesis investigates some coefficient functionals and certain estimates of several subclasses of univalent functions like Sakaguchi starlike functions, Sakaguchi convex functions, close-to-convex functions, MacGregor Analytic functions and the Big q-Bessel functions. We study sharp estimates on certain coefficients like initial coefficients, inverse coefficients, logarithmic coefficients, logarithmic inverse coefficients, moduli difference of successive coefficients and coefficient functionals like Hankel determinants, Toeplitz determinants, Hermitian-Toeplitz determinants, Zalcman functionals, Fekete--Szeg\H{o} functional, Pre-Schwarzian norm estimates and Higher-order Schwarzian derivatives for such subclasses. Further, we discuss Growth and Distortion theorem for a subclass of close-to-convex functions. Moreover, we determine the radius of k-parabolic starlikeness, the radius of k-uniform convexity of order α $(0 \leq \alpha <

1)$, the radius of lemniscate starlikeness and the radius of lemniscate convexity for the class of the Big q-Bessel functions.

Contents

1. Introduction 2. Close-to-Convex Functions 3. Sakaguchi Functions 4. Sakaguchi Functions with Missing Second Coefficient 5. MacGregor Analytic Functions 6. The Big q-Bessel Functions. List of Publications. References. AMS MSC 2020 Classification and Keywords.

10. SHARMA (Praveen)
Nonlinear Interpolation of Functions and Applications to Nonlinear Evolution Equations.
Supervisors: Prof. Sachi Srivastava and Prof. Ralph Chill
Th 29159

Abstract

The thesis entitled \emph{Nonlinear Interpolation of Functions and Applications to Nonlinear Evolution} develops a unified framework for real interpolation of functions on Banach spaces and applies it to establish regularity results for nonlinear evolution equations governed by m-accretive operators, such as the\emph{q-Laplace operator} and \emph{the porous medium operator}.

Contents

1. Introduction 2. Interpolation of functions 3. Interpolation associated with m-accretive operators 4. Regularizing semigroups 5. Reiteration theorems 6. Weighted L^2-Maximal Regularity for Subgradients.