CHAPTER 38

MATHEMATICS

Doctoral Theses

01. AGGARWAL (Amita)
Toeplitz Type Operators and Hyponormality on the Bergman and Fock Spaces.
Supervisor: Prof. Anuradha Gupta
Th 28376

Abstract

This thesis aims to investigate the Toeplitz operators and their generalizations to certain analytic function spaces. The thesis explores the hyponormality of Toeplitz operators linked with various symbols on the Bergman space and the Fock space. We demonstrate the hyponormality of the Toeplitz operator on the Fock space when the symbol is a monomial non-harmonic function and observe that the sum of two hyponormal Toeplitz operators may not necessarily retain hyponormality. The sufficient conditions for hyponormality of the sum of two Toeplitz operators corresponding to various non-harmonic symbols on the Fock space are also derived. Further, the thesis presents the characterizations of quasi-homogeneous Toeplitz operators of positive degree as hyponormal. An alternative approach to examine the hyponormality of Toeplitz operators corresponding to monomial non-harmonic symbols through quasi-homogeneous polynomials is also given. The study of H-Toeplitz operators explores its algebraic properties and provides a comprehensive matrix representation tailored to harmonic symbols. The necessary conditions for the co-hyponormality of the operators are also examined. The necessary and sufficient conditions for the commutativity of H-Toeplitz operators corresponding to the analytic symbols are presented. Further, the concept of the generalized slant H-Toeplitz operator is introduced on the Bergman space, focusing on obtaining its matrix representation corresponding to harmonic symbols. We also identify invariant subspaces and study the Berezin transform for the generalized slant H-Toeplitz operators.

Contents

02. BANSAL (Piyush)
Uncertainty Principles for Certain Integral Transforms on Unimodular Groups of Type I.
Supervisors: Dr. Ratikanta Panda and Prof. Ajay Kumar
Th 28377

Abstract

Uncertainty principle is almost a century old concept introduced to gain knowledge of the simultaneous limits of precision of a particle`s position and momentum. The

very first uncertainty principle, named as Heisenberg uncertainty principle, was introduced in 1927 by a German physicist Werner Heisenberg. He proposed that it is not possible to accurately measure the position and momentum of a microscopic particle simultaneously. If we try to find the accurate position of the particle, the information about its momentum becomes less accurate and vice versa. For the past many decades, the uncertainty principles have been deeply studied both quantitatively and qualitatively. In this thesis, we have studied Heisenberg uncertainty inequality, Pitt`s inequality, logarithmic uncertainty inequality, Nazarov uncertainty inequality and qualitative uncertainty principle (QUP) for various integral transforms such as the Fourier transform, the Gabor transform, the shearlet transform, the wavelet transform and the continuous modulated shearlet transform (CMST) on a variety of unimodular groups of type I. In first chapter, we start with some basic notations and terminology. We shall then discuss the representation theory of locally compact groups, Fourier transform, Plancherel theorem and some related concepts. Some well-known transforms are summarized. The explicit forms of the uncertainty principles such as Heisenberg uncertainty inequality, Pitt`s inequality, logarithmic uncertainty inequality, Nazarov uncertainty inequality and QUP have been given for Fourier transform on R^n. Chapter 2 discusses Pitt`s inequality, logarithmic uncertainty inequality and Heisenberg uncertainty inequality for exponential solvable Lie groups with non-trivial center, connected nilpotent Lie groups with non-compact center, the Heisenberg motion group and the diamond Lie groups. In Chapter 3, we discussed CMST that generalizes the well-known transforms such as the Gabor transform, the wavelet transform and the shearlet transform. Some important properties like Plancherel formula and inversion formula are obtained. Heisenberg uncertainty inequality for CMST has also been proved. The chapter 4 includes the discussion of QUP for CMST on several classes of groups including abelian groups, compact extensions of abelian groups and the Heisenberg group. A variation of QUP for CMST has been proved in the last section of this chapter. In the last chapter, we focus on Nazarov uncertainty for the Fourier transform and the CMST on the groups of the form R^n x K (K being a locally compact group) and Euclidean motion group.

Contents

03. BRIJ MOHAN
Generalization of Equations, Solutions, and Techniques for KdV-type Evolution Equations.
Supervisor: Prof. Raj Kumar
Th 28717

Abstract

This thesis develops and applies novel analytical and symbolic techniques to investigate integrable higher-dimensional nonlinear KdV-type evolution equations. It spans kink-solitons, generalized soliton solutions, higher-order rogue waves, and dispersive solitons with new insights into their dynamics, parameter dependencies, and graphical representations. We extend the direct method given by Hirota to a symbolic bilinear technique, which broadens the truthfulness of exact solutions to generalized solutions with arbitrary parameters. Across the chapters, multiple new integrable equations are proposed, tested for Painleve integrability, and transformed into bilinear or trilinear forms for the construction of solutions. In this research, we

formulate novel integrable equations and investigate these equations by new techniques. We propose a direct symbolic approach and a symbolic bilinear technique to explore rogue waves and generalized soliton solutions, respectively. With the usefulness of the Cole-Hopf transformation, we formulate the bilinear and trilinear forms that enable the systematic construction of solitons, kink waves, dispersive solitons, and rogue waves. Furthermore, the proposed novel symbolic bilinear technique generalizes Hirota's bilinear method by incorporating arbitrary parameters. This research contributes to extending the exploration of integrable models in soliton theory. Also, it establishes new methodological pathways for generating exact solutions of higher-dimensional nonlinear partial differential equations.

Contents

04. DIMPI
Fixed Point Sets and Orbit Spaces of Involutions on Some Finite CW-Complexes.
Supervisor: Dr. Hemant Kumar Singh
Th 28378

Abstract

Let G be a compact Lie group and X a finite CW-complex. The study of the cohomological structure of the fixed point sets and orbit spaces of an action of group G on space X has been extensively explored by many mathematicians. In continuation, this thesis focuses on determining the possible fixed point sets of actions of a cyclic group of prime order or the circle group, on various finite CW-complexes such as arbitrary wedges of two / three spheres, products of projective spaces and spheres, and products of three spheres. The remaining part of the thesis examines the possible orbit spaces of free actions of G = Z_p, where p is prime, or G=S^d with d=1,3, on certain spaces like arbitrary wedges of three spheres, products of projective spaces and spheres, or products of three spheres. The results of the thesis generalize several previously known results. As an application, we have derived Borsuk-Ulam type results.

Contents

05. GEETA
Composition and Generalized Weighted Composition Operators on Certain Analytic Function Spaces.
Supervisor: Prof. Anuradha Gupta
Th 28379

Abstract

The thesis includes the study of composition operators and generalized weighted composition operators on various Hilbert spaces of analytic functions like weighted Bergman spaces, general weighted Hardy spaces, Fock-type spaces and vector-valued Bergman space. Schwartz's well-known result which establishes the important relationship between almost multiplicative operator and composition operators on Hardy spaces, has been generalized to the weighted Bergman spaces. The necessary and sufficient conditions for the product of a composition operator with the adjoint of a composition operator to be isometric on weighted Bergman spaces are provided. We discuss how composition operators on derivative Hardy space behave differently from Hardy space and weighted Bergman space. We establish the necessary and sufficient conditions for the composition operator to be an isometry on general weighted Hardy spaces and vector-valued Bergman space. Also, characterization of self-adjoint and co-isometric composition operators is determined on general weighted Hardy spaces, Fock-type spaces and vector-valued weighted Bergman spaces. For linear fractional self-map on the open unit disk, we obtain characterization for both posinormal and coposinormal composition operators on vector-valued Bergman space. We obtain upper and lower bounds for the norm of composition operators on vector-valued weighted Bergman spaces. Weighted composition operators which are generalization of composition operators, are also explored. Self-adjoint and co-isometric weighted composition operators are discussed on general weighted Hardy spaces and Fock-type spaces. Moreover, generalized weighted composition operator is also studied. The necessary and sufficient conditions for a generalized weighted composition operator to be bounded under certain assumption are discussed on general weighted Hardy spaces and vector-valued weighted Bergman spaces. Also, the characterization for Hilbert-Schmidt generalized weighted composition operator on general weighted Hardy spaces is obtained.

Contents

06. GUPTA (Sakshi)
Scalarization and Well-Posedness in Vector and Set Optimization.
Supervisor: Prof. Manjari Srivastava
Th 28380

Abstract

While tackling optimization problems, we frequently encounter with multiple conflicting objectives. Such problems are known as multiobjective or vector optimization problems. Also, there are optimization problems where the objective function is a set-valued mapping and are referred to as set optimization problems. These problems find widespread application in various fields, including mathematical finance, robust optimization, duality principles, welfare economics, multiobjective optimization, and fuzzy programming. This thesis explores two key concepts in vector and set optimization problems: scalarization techniques and well-posedness. These concepts are examined using different set order relations. In optimization theory, the concept of well-posedness is fundamental to the study of stability theory. This has led many researchers to investigate well-posedness across

various types of optimization problems, including vector optimization problems, variational inequality problems, set optimization problems, parametric set optimization problems, and more. Scalarization is a traditional method used to describe minimal solutions of vector or set optimization problems by relating them to minimal solutions of a corresponding scalar problem. Commonly employed nonlinear scalarizing functions include the Gerstewitz function, the oriented distance function, and their generalizations. This thesis focuses on exploring different scalarization schemes in vector and set optimization through these nonlinear scalar functions.

Contents

07. KRISHAN PAL

Stokes Drag Effect in the Restricted Four- Body Problem with Different Perturbations and Variable Mass.

Supervisor: Dr. Amit Mittal

Th 28719

Abstract

In this thesis, we have studied the effect of Stokes drag, oblateness/prolateness, radiation pressure and Coriolis and centrifugal forces in the restricted four-body problem with variable mass. This thesis deals with the existence, locations and stability of the equilibrium points as well as zero velocity curves (ZVCs) under the effect of Stokes drag, oblateness, radiation pressure and Coriolis and centrifugal forces in the restricted four-body problem (R4BP) with variable mass. The equations of motion of the infinitesimal body (fourth body) have been investigated by using the Jeans` law and space time transformation of Meshcherskii. The effect of dissipative constant k, radiation parameter q, oblateness parameter A, constant coefficient due to variable mass α, perturbations due to Coriolis and centrifugal forces β1 and β2 parameters on the locations of the libration points and their stability are investigated. The numerical investigation reveals that all the equilibrium points are non-collinear and the collinear equilibrium points do not exist due to the presence of Stokes drag. The linear stability of the equilibrium points has been investigated. It is observed that all the equilibrium points are unstable for all values of the parameters used. Also, it has been observed that the regions of motion increase for the increasing values of the parameters A, q and α whereas these regions decrease for the increasing values of the dissipative constant k. Finally, we have justified the significance of our work with the real system by applying it to an appropriate astrophysical system.

Contents

08. MARIG (Sawan Kumar)
Basins of Convergence and Stability of Equilibrium Points in the Modified Magnetic-Binary Systems.
Supervisor: Prof. Vinay Kumar
Th 28718

Abstract

The Restricted three-body problem (RTBP) models the motion of a small body, P_3, under the gravitational influence of two primary bodies, P_1 and P_2, which orbit around their common centre of mass. The mass of P_3 is assumed to be negligible, ensuring it does not affect the motion of primaries. This work extends the classical RTBP framework by incorporating magnetic effects, modelling the primaries as magnetic dipoles, thus introducing the magnetic-binary problem, as first proposed by \citeauthor{mavraganis1978magnetic} \citeyear{mavraganis1978magnetic}. The research focuses on analysing the dynamics of P_3 under the combined influence of gravitational and magnetic fields. This study examines the equilibrium points, their stability and the basins of convergence within this extended system. It explores the effects of variable mass, Yukawa forces, modified gravitational forces and perturbations in Coriolis and centrifugal forces on the dynamical system. The topological structure of the basins of convergence exhibits complex, often fractal configurations that vary significantly with the change in physical properties and initial conditions of the system. Numerical methods are employed to investigate how these properties influence the stability of equilibrium points and the overall dynamics. Chapter 1 introduces necessary definitions, technical terms, and a comprehensive review of the literature on the Restricted Three-Body Problem (RTBP) and the magnetic binary problem. It outlines methodologies for investigating the existence, stability and regions of possible motion in dynamical systems. The chapter also introduces the multivariate Newton-Raphson iterative scheme used to analyse the topology of basins of convergence and identify fractal structures using the log 2 criterion, laying the foundation for developing the research problem.

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09. MEGHNA
Starlikeness of Certain Carathéodory and Special Functions.
Supervisor: Dr. Naveen Kumar Jain
Th 28807

Abstract

Geometric Function Theory is a branch of Complex analysis that studies the geometric properties of analytic functions. Univalent functions are one of the major parts in study of this theory since they don't overlap or fold the domain onto itself, making it easier to study the geometry of the image domains. A special class of univalent functions is that of starlike functions. A domain in the complex plane is starlike with respect to the origin

if every straight line drawn from the origin to a point inside the domain lies completely inside it. A function is called starlike if the region it maps the unit disc onto has this property. This thesis investigates the starlikeness and related geometric properties of various analytic functions using various tools such as differential subordination, admissibility conditions and the study of special functions. The study begins with Carathéodory functions, which are analytic in the unit disc and have positive real part. One approach used to show that certain Carathéodory functions are starlike with respect to different subclasses is differential subordination. The results obtained are sharp and illustrated graphically. When differential subordination fails to give the sufficient conditions for starlikeness, another technique called admissibility conditions is applied. This method works with implications involving the function and its higher derivatives, ensuring that one function stays within the domain of another. The results obtained gives certain bounds on the parameters under which functions belong to subclasses of starlike functions. Inspired by the Bieberbach conjecture, the study introduces a new class of quotient functions with fixed second coefficients. For this class, the growth and distortion theorems are proved. In addition, radius estimates for this class are studied. In another part of this work, various special functions and their power series expansions are utilized to prove the starlikeness of analytic functions. Further, the concept of admissibility is used to establish second and third order differential subordination relations between the analytic function and functions associated with the right half-plane. The last part of the work uses the disk method to study starlikeness of special functions. The monotonicity of the Miller Ross function is established and conditions are found so that their solutions belong to Janowski starlike classes. As an application, the method is applied to special functions such as Bessel functions, Struve functions and Coulomb wave functions, showing how they belong to certain class of starlike functions. In conclusion, this thesis extends the theory of starlike functions by giving new sufficient conditions, creating new subclasses, and providing sharp bounds. It brings together different kinds of functions-Carathéodory functions, quotient functions, and special functions under one framework.

Contents

10. MONIKA

Analytical Solutions and Dynamics of Various Waveforms of Specific Multidimensional Nonlinear Evolution Equations.

Supervisor: Dr. Sachin Kumar

Th 28381

Abstract

In this thesis, we analyze nonlinear evolution equations (NLEEs), which are fundamental in understanding various physical phenomena. Our focus is on deriving analytical solutions for specific NLEEs, including the (1+1)-dimensional Perturbed Chen Lee-Liu equation, (1+1)-dimensional cubic-quintic nonlinear Schrödinger equation, (2+1)-dimensional Heisenberg ferromagnetic spin chain equation, (2+1)-dimensional Generalized Benjamin-Ono equation, (2+1)-dimensional Breaking soliton equation and the (4+1)-dimensional Fokas equation. We apply several analytical methods, such as the generalized Riccati equation mapping method, the (G`/G)-expansion method, the Unified method, the inverse (G`/G)-

expansion method, and the generalized exponential rational function method. These techniques yield solutions in the form of exponential, trigonometric, and hyperbolic functions forms. The physical interpretations of these solutions reveal various wave patterns, shedding light on phenomena such as solitons and other wave structures.

Contents

11. NIKITA

Exact Solutions and Dynamical Analysis of Certain Non-Linear Evolution Equations Using Analytical Methods.

Supervisor: Prof. Sachin Kumar

Th 28382

Abstract

This thesis focuses on the analytical investigation of nonlinear evolution equations, which are fundamental in describing complex physical processes such as wave propagation, nonlinear interactions, and fluid dynamics. These equations, as a subset of nonlinear partial differential equations (PDEs), exhibit intricate dynamics due to inherent nonlinearities, making their analysis both challenging and significant. The primary objective of this research is to employ a variety of analytical techniques, supported by dynamical systems theory, to derive exact solutions and examine qualitative behaviors of selected nonlinear models. Five representative equations are studied: the (3+1)-dimensional Vakhnenko–Parkes equation, the (1+1)-dimensional Salerno equation, the Painlevé-integrable (3+1)-dimensional nonlinear evolution equation, the (2+1)-dimensional Konopelchenko–Dubrovsky system, and the Kuralay-II equation. For these models, advanced solution strategies such as the generalized Kudryashov method, the generalized exponential rational function method (GERFM) and its extended variant, the generalized Riccati equation mapping method, the sinh–Gordon equation expansion method, and the unified method are systematically applied. These approaches yield a wide spectrum of exact solutions, including solitons, periodic structures, kinks, lump-type waves, and multi-soliton interactions. Beyond solution construction, dynamical systems theory is utilized to classify solution behaviors, investigate equilibrium structures, and explore bifurcation phenomena. Special emphasis is given to perturbed systems, where quasi-periodic and chaotic dynamics are identified through detailed phase portrait analysis. The graphical representations of obtained results provide deeper insight into the amplitude structures, stability, and complex dynamical patterns inherent in nonlinear systems. Overall, this work contributes new families of exact analytical solutions, extends existing methodologies, and offers comprehensive dynamical analyses of nonlinear evolution equations. The results not only enrich the mathematical theory of nonlinear PDEs but also enhance their applicability to diverse physical contexts.

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Painlevé-Integrable (3+1)-dimensional nonlinear evolution equation 5. Exact solutions and dynamical analysis of the (2+1)-dimesnional Konopelchenko-Dubrovsky system 6. Dynamical analysis of the integrable Kuralay equation. Remarks and Future scope, List of Publications and References.

12. SABHARWAL (Anuradha)
Association Schemes Over Some Algebraic Structures and Their Applications in Cryptography.
Supervisor: Dr. Pooja
Th 28720

Abstract

Association schemes play a central role in algebraic combinatorics and have wide applications in coding theory, design theory, and finite geometry. As combinatorial abstractions of group actions, they provide a strong algebraic foundation for constructing secure systems. While most existing studies emphasize symmetric and commutative association schemes arising from conjugacy classes of finite groups, these are restrictive for cryptographic applications. This thesis develops non-symmetric association schemes over diverse algebraic structures and investigates their use in cryptography, with a unified framework for encryption and secure transmission. Chapter 1 introduces fundamental concepts of association schemes together with an overview of cryptography and the statistical metrics such as correlation coefficients, PSNR, and MSE that are used for performance evaluation. Chapter 2 presents new constructions of non-symmetric association schemes on dihedral, symmetric, and alternating groups, extending further to matrix groups and finite group rings. Chapter 3 demonstrates how classical ciphers naturally arise from association schemes and introduces three new constructions namely the Modified Playfair Cipher using Association Schemes (MPCAS), the Extended Association Scheme Cipher (EASC), and the Random Matrix Transformation on Association Schemes (RMTAS). In the next two chapters the developed association schemes and ciphers are applied to real data. Chapter 4 focuses on secure systems for text and image data. A crypto-steganographic method is proposed which combines association schemes with random matrix transformations and Arnold scrambling to embed encrypted data into cover images, ensuring imperceptibility and robustness. Chapter 5 extends the framework to secure graphs and videos by combining RMTAS with Arnold transforms. Comparative analysis with existing methods supported by experimental statistics demonstrates superior imperceptibility, robustness, and computational efficiency. Overall the thesis establishes association schemes as a versatile tool for cryptography and secure multimedia transmission while opening directions for quantum-resistant cipher design.

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13. SETU RANI
Symmetry Reductions and Analytical Solutions of Certain Non-Linear Evolution Equations.
Supervisor: Prof. Raj Kumar
Th 28721

Abstract

This thesis investigates the application of advanced analytical methods to nonlinear evolution equations (NLEEs), which play a critical role in modeling phenomena such as wave propagation, fluid dynamics, and nonlinear interactions in various physical media. These NLEEs are a subclass of nonlinear partial differential equations (PDEs) that characterize time-dependent processes involving complex nonlinear relationships. Due to their inherent nonlinearity, such equations pose considerable challenges for analytical treatment, unlike their linear counterparts. This research focuses on deriving exact solutions and symmetry reductions using a combination of Lie symmetry analysis, optimal system theory, and other nonlinear analytical techniques. The thesis applies these methods to five important NLEEs: the (2+1)-dimensional Sakovich equation, the (1+1)-dimensional coupled Boussinesq–Burgers system, the (2+1)-dimensional weakly coupled B-type Kadomtsev–Petviashvili (BKP) system, the (2+1)-dimensional dissipative long wave (DLW) system, and the (1+1)-dimensional complex Kraenkel–Manna–Merle (cKMM) system. Chapter 1 lays the theoretical groundwork, introducing Lie group transformations, continuous symmetry theory, and several analytical methods including the generalized Kudryashov method, the modified auxiliary equation method, and the sinh-Gordon expansion method. It also briefly discusses bifurcation theory and chaotic dynamics. Chapter 2 applies Lie symmetry and optimality conditions to the Sakovich equation, yielding various solitary wave solutions through symmetry reduction. Chapter 3 studies the BB system using a blend of Lie symmetry and Kudryashov's method, resulting in multiple exact solutions including bell-shaped and periodic structures. Chapter 4 explores the weakly coupled BKP system, deriving new generalized solutions with arbitrary functional parameters and visualizing them through 2D and 3D plots. Chapter 5 examines the DLW system, producing diverse wave profiles such as kink-type and periodic solitons, validated through both analytical and numerical methods. Finally, Chapter 6 analyzes the cKMM system using two novel methods, revealing exact trigonometric, hyperbolic, and elliptic solutions, along with a detailed examination of bifurcation and chaotic behavior. Overall, this work emphasizes the power of symmetry-based and analytical methods in simplifying complex nonlinear PDEs and constructing rich families of exact solutions, which not only deepen the theoretical understanding but also offer practical insights into the dynamics of nonlinear systems.

Contents

14. SHARMA (Abhishek)
Numerical Solutions and Convergence Analysis of Fractional Order Differential Equations with Bounded Delay.
Supervisors: Dr. Surendra Kumar and Dr. Harendra Pal Singh
Th 28383

Abstract

Differential equations play a prominent role in many disciplines such as mathematical physics, biology, engineering, among others. The delay differential equations (DDEs) and the fractional differential equations (FDEs) are highly applicable. In this thesis, numerical solutions of fractional delay differential equations (FDDEs) are studied using block boundary value methods (BBVMs), Lagrange interpolating polynomial and the Haar wavelet method (HWM). The fractional derivative used in FDEs is taken in the Caputo sense. This thesis consists of five chapters followed by a list of references at the end. Chapter 1 serves preliminaries and forms a base for the other chapters. In Chapter 2, a new numerical scheme termed as fractional delay block boundary value method (FDBBVM) is used to find numerical solutions of FDDEs. The scheme FDBBVM is shown to be convergent and globally stable. Chapter 3 presents a new numerical scheme, namely, the fractional multi-delay block boundary value method, which is used to solve FDEs with multiple delays. The proposed scheme is globally stable and convergent. The numerical solutions of FDDEs are investigated in Chapter 4. A constant time delay is taken in the Caputo derivative of the state variable and also in the state variable. To obtain a solution, an extension of BBVM is used, which is convergent and globally stable. In Chapter 5, the numerical solutions of FDEs with multiple delays are obtained using the HWM and the Lagrange interpolating polynomial. Afterward, the problem is generalized to a system of nonlinear FDEs with piecewise continuous arguments and multiple delays. The HWM is also extended for the system of nonlinear FDEs, and it is verified that these numerical schemes are convergent. The computational efficiency and accuracy of all the proposed numerical schemes are shown via several illustrations.

Contents

15. SHARMA (Deepika)
Discontinuous Waves and Riemann Problem for Extended Class of Chaplygin Gases.
Supervisor: Dr. Randheer Singh
Th 28722

Abstract

Many physical phenomena observed in the universe are essentially nonlinear and often modelled by quasi-linear partial differential equations (PDEs). This thesis presents quasi-linear hyperbolic systems of PDEs that are governed by two-phase flow model and macroscopic production model along with the extended class of Chaplygin gases. This thesis is apportioned into five chapters. Chapter 1 serves as the foundation of the thesis by introducing the preliminaries, literature survey and thesis layout. In Chapter 2, a novel modified Chaplygin equation of state for non-ideal two-phase flow is derived and one-dimensional isentropic non-ideal modified Chaplygin two-phase flow model comprising a source term is presented. The model is reduced to an equivalent system of ordinary differential equations (ODEs) via Lie group scheme. The transport equation for the singular surface is derived and solved

with the system of ODEs. Further, the effects of source term and non-idealness on singular surface are investigated. In Chapter 3, the two-phase flow model with modified Chaplygin gas consisting of source term is reduced to a system of ODEs via Lie symmetry analysis. Further, the evolutionary law for the characteristic shocks is derived and solved using an exact solution. The patterns of flow variables and the effects of source term on the characteristic shock are analyzed. In Chapter 4, the effects of non-idealness and source term on the characteristic shock and acceleration wave for the PDEs describing two-phase real modified Chaplygin flow model with source term, are investigated. The amplitude of the reflected and transmitted wave generated after interaction are computed. Chapter 5 deals with the formation of delta shock waves and vacuum state, respectively, in the Riemann problem for the pressureless macroscopic production model. The Riemann solutions for the two distinct macroscopic production model with Umami Chaplygin gas, under the suitable flux-approximation are obtained during limiting process.

Contents

16. SUMIT
Relative Versions of Some Selection Principles in Topological Spaces.
Supervisors: Dr. Hemant Kumar Singh and Prof. B. K. Tyagi
Th 28384

Abstract

Selection principles in topology deal with the systematic selection of elements from sequences of open covers to satisfy certain covering properties. The aim of the thesis is to explore the relative versions of various selection principles. Selection principles play a crucial role in understanding the combinatorial as well as topological properties of spaces and their relative versions provide a refined perspective by considering covers that interact with the subspaces. The main investigation of relative selection principles was commenced by Kočinac in 2000. We introduce and examine the notion of set strongly 1-starcompact spaces. We provide a variety of examples to understand the relationship of set strongly 1-starcompactness property with other set versions of selective covering properties. Further, we study the preservation properties of these spaces under subspaces and various types of mappings. We introduce and analyze the notions of relatively* SM, relatively* SSM, relatively* SH, relatively* SSH, relatively* SKM and relatively* SKH properties. Our investigation focuses on the relationships between these relative versions, their behaviour in subspaces and their preservation under various types of mappings. Also, we explore the relative versions of star-Menger property introduced by Bonanzinga et al. in 2007. We provide a variety of examples to understand their relationships with other relativizations of star selection principles. Moreover, we investigate the behaviour of these spaces under various types of mappings. Furthermore, we introduce the notion of countably ρI-compact ideal topological spaces. We investigate their preservation properties under subspaces and mappings. Moreover, we establish several characterizations of this notion.

Contents

17. TAHU (Biswajit)
Analysis of Well-Posedness and Duality for Set Optimization Problems.
Supervisor: Prof. Pankaj Kumar Garg
Th 28723

Abstract

In this thesis, we mainly focus on studying the well-posedness, well-setness and duality theory for set and parametric set optimization problems. The theory of well-posedness ensures stability, existence and continuous dependence of solutions in an optimization problem. The theory of well-setness plays a crucial role in ensuring that a set optimization problem is well-posed and that its solutions are meaningful, stable and reliable. The work done in this thesis is a part of a broader effort to extend the classical optimization theories to more general and complex set optimization frameworks. Lagrangian duality is a powerful tool in optimization theory. It involves transforming a constrained optimization problem into a dual problem, which often simplifies the problem and can offer insights into the structure of the original problem. It lays theoretical foundations for optimization algorithms in modern contexts like game theory, machine learning and operations research. The thesis consists of four chapters with the following details. In Chapter 1, entitled "Introduction", atfirst we present the historical background of 'Optimization Problem' and provide some of their applications in real life. In the Chapter 2, we introduce various notions of Levitin--Polyak well-posedness for parametric set optimization problem concerning Pareto efficient solutions. The key idea was to adapt the Levitin-Polyak condition to parametric set optimization settings. The work is highly applicable in multi-objective decision-making, economics, engineering design and control theory, where decisions involve sets of outcomes rather than single values. The theory established in the Chapter 3 is crucial when dealing with real-world problems where the data or parameters are often uncertain and the optimization models needs to handle small variations without leading to drastic changes in the solutions. By introducing a metrical framework for studying set optimization, the work potentially extends existing theories in optimization, especially in dealing with parametric and set valued functions. This could open doors for new algorithms or methods for solving optimization problems in fields where uncertainty or parameter variation is a significant concern. The research done in the Chapter 4 is highly relevant in advancing the theory of set optimization by introducing the concept of Lagrangian duality in the context of set optimization problems, particularly through the use of improvement sets. Thus, the work done in this thesis is not just theoretical but has broad, practical relevance across many contemporary technological and scientific fields.

Contents

18. UPADHYAY (Anjali)
Existence of Mild Solutions and Stability Analysis of Stochastic Differential Systems.
Supervisor: Dr. Surendra Kumar
Th 28385

Abstract

Systems exhibiting significant uncertainty or randomness are often modeled using stochastic differential equations. Due to the non-local nature of fractional derivatives, fractional differential equations provide a more accurate mathematical description of several phenomena. This thesis mainly studies the concepts related to mild solutions and the stability of different types of stochastic functional differential equations and inclusions. This thesis consists of five chapters, followed by a list of references at the end. The chapter-wise sketch of this thesis is as follows: Chapter 1 is introductory and includes basic definitions and results, which are required to study the rest of the thesis easily. It begins with a brief historical overview of the subject and then presents preliminary results of stochastic analysis, fractional calculus, semigroups of linear operators, and various valuable inequalities. Chapter 2 considers multi-term fractional stochastic differential inclusions. An expression for a mild solution of the considered system is obtained with the help of a family of resolvent operators. The successive approximation method (SAM), in combination with the stochastic analysis, determines the existence and uniqueness of the solution. Furthermore, some sufficient conditions are given to ensure the exponential decay of the mild solution. The results are derived under non-Lipschitz conditions. An example is provided to validate the results produced in this chapter. In Chapter 3, we explore a fractional non-autonomous stochastic impulsive differential system. The notion of a mild solution is defined via the operators induced by the closed linear operators and the probability density function. The existence of a solution is proved by SAM. Additionally, the Ulam-Hyers stability of the considered system is studied. The results are derived under the Lipschitz conditions. Finally, an example is discussed. Chapter 4 contains two sections. In the first section, a fractional non-autonomous stochastic differential system is studied. The existence of a mild solution is established via SAM. Furthermore, we discuss the Ulam-Hyers stability of the considered system. The findings are established under Lipschitz conditions. The second section examines the existence and sensitivity of a mild solution for fractional non-autonomous stochastic differential inclusions. The obtained results are proved under non-Lipschitz conditions. At the end, two illustrations are demonstrated to validate the results obtained in this chapter. Chapter 5 deals with a class of second-order non-autonomous stochastic impulsive differential inclusions with unbounded delay. The existence of a solution is deduced by employing a fixed point strategy for a set-valued map together with the evolution operators. Finally, an example is analyzed for theoretical developments.

Contents

19. VIKESH KUMAR

Embedding of Semitopological Groups into the Product of Some Families of Semitopological Groups.

Supervisors: Dr. Hemant Kumar Singh and Prof. Prof. Brij Kishore Tyagi

Th 28386

Abstract

The aim of this thesis is to embed semitopological groups into the product of some families of semitopological groups. It consists of five chapters. In Chapter 1, we discusses historical developments, basic notions and results of general topology and topological groups. Chapter 2 is devoted to the notion of ωs-balanced semitopological groups. Using the concept of ωs-balancedness and some cardinal functions, we embed semitopological groups into the product of some families of strongly developable semitopological groups. The purpose of Chapter 3 is to deal with the concept of weakly ω-balanced semitopological groups and the embedding of a semitopological group into the product of some families of weakly developable semitopological groups. In Chapter 4, we define θ-ω-balanced and $\delta$$\theta$-$\omega$-balanced semitopological groups. We embedded regular (Hausdorff, $ T_{1} $) semitopological groups as subgroups into the product of regular (Hausdorff, $ T_{1} $) semitopological groups with θ-base ($\delta$$\theta$-base). Further, we obtain a characterization of a subgroup of product of regular (Hausdorff, $ T_{1} $) quasi-developable semitopological groups. Chapter 5 is concerned with two notions: property (τ^{*}) and para τ-discrete spaces. We show that the class of semitopological groups having property (τ^{*}) is closed under subgroups and products. Also we show that the class of metric spaces is contained in the class of para τ-spaces. we discuss the relationship between property (τ^{*}) and para τ-discrete semitopological groups. We also give necessary and sufficient conditions under which a $ T_{1} $ semitopological group can be embedded as a subgroup of product of para τ-discrete semitopological groups of character less than or equal to τ.

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20. VIRENDRA KUMAR

Study of the Effect of Resistive Force and Earth`s Equatorial Ellipticity on Resonant Curves using Unperturbed Solution of Motion of Geo-Centric Satellite.

Supervisor: Prof. Sushil Yadav

Th 28387

Abstract

This thesis deals with two-body, three-body and four-body problems including the effect of Earth`s equatorial ellipticity and resistive force on the resonant curves. We have determined the equations of motion of the Geo-centric satellite with the help of Earth's potential in the spherical coordinate system, and obtained its unperturbed solution. Using the unperturbed solution, we have found the resonant points in the satellite`s motion and investigated the resonant curves due to the ratio of two frequencies and

oscillatory amplitudes. On the resonant curves, we have analyzed the effect of orbital elements, Earth`s equatorial ellipticity and coefficients of resistive force. It is observed that the oscillatory amplitude increases if eccentricity increases, and in case of increasing the semi-major axis, the oscillatory amplitude decreases. It is also observed that the oscillatory amplitude decreases if the coefficient of resistive force increases.

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21. YADAV (Akshay Ankush)
On Generation of Novel Classes of Permutation Polynomials over Finite Fields.
Supervisors: Prof. Arvind and Dr. Indivar Gupta
Th 28808

Abstract

Permutation polynomials (PPs) over finite fields are an important object of study due to their algebraic richness and wide applications in cryptography, error-correcting codes, and combinatorial designs. A polynomial $f(x)\in Fq$[x] is a permutation polynomial if it defines a bijection on Fq. Over the years, several families such as binomials, linearized polynomials, Dickson polynomials, and cyclotomic classes have been systematically explored. However, significant challenges remain in constructing new families and classifying them under suitable equivalence relations. In particular, the problem of determining permutation trinomials and quadrinomials over quadratic extensions of finite fields, and establishing their novelty under quasi-multiplicative (QM) equivalence, remains difficult and largely unresolved. This thesis makes contributions in two directions. First, we construct infinite families of permutation trinomials over F_2^2m and both trinomials and quadrinomials over F_5^2n. Using algebraic tools such as the AGW criterion, root-of-unity analysis, and trace functions, explicit conditions are derived to ensure their permutation properties. These constructions are rigorously verified to be QM-inequivalent to previously known classes, thereby establishing their novelty. Second, we propose a general and efficient algorithm for testing QM-equivalence among permutation trinomials. This framework resolves limitations of earlier ad hoc methods and provides a systematic tool for equivalence classification. Its application not only confirms the novelty of the proposed families but also corrects misclassifications reported in earlier literature. The study represents progress toward a broader program of classifying permutation polynomials over finite fields. While the focus is on fields of characteristic two and five, the developed methods are adaptable to other characteristics and higher-degree polynomials. The constructions and algorithms introduced here thus provide a solid foundation for further research in algebra, finite field theory, and their applications.

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22. YOGESH KUMAR
Algebraic and Combinatorial Study of Cryptographically Significant MDS Matrices.
Supervisors: Prof. Atul Gaur and Dr. P. R. Mishra
Th 28388

Abstract

This thesis develops new algorithms, combinatorial counts, and structural characterizations of Maximum Distance Separable (MDS) matrices with various algebraic properties over finite fields, advancing both theory and implementation. First, we present an improved algorithm for computing the branch number of non-singular matrices that refines the classical method while reducing its computational complexity. A detailed comparative analysis with the standard approach is provided. Second, we propose two hybrid algorithms that generate all n × n MDS and involutory MDS matrices over F_p^m. By introducing a representative matrix form, the search is systematically reduced to (n − 1) × (n − 1) MDS matrices. We provide an explicit formula for the count of all 3 × 3 MDS matrices over F_2^m and explicitly enumerate all 4 × 4 cases for m = 3, 4. Third, we propose a new matrix form to construct involutory MDS matrices of even order. For the 4 × 4 case over F_2^m, our method reduces the effective search space to (2^m-1)^5 class-representative matrices, enabling exact enumeration for m=3,4...,8. Fourth, we investigate semi-involutory and semi-orthogonal matrices, presenting two algorithms for verifying these properties for n×n matrices over F_p^m. Circulant matrices with these properties are also explored. We provide explicit formulas for counting 3 × 3 semi-involutory, orthogonal, and semi-orthogonal MDS matrices over F_2^m and provide exact counts for the 4 × 4 case over F_2^m for m = 3, 4. Finally, we present the construction of two Hadamard matrix forms over F_2^m to generate 4 × 4 and 8 × 8 involutory MDS matrices. The first yields 4 × 4 matrices directly, while the second supports efficient hybrid construction of 8 × 8 matrices. We carry out a comparative study based on the XOR count on constructed MDS matrices with MDS matrices available in the literature.

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