CHAPTER 30

MATHEMATICS

Doctoral Theses

01. ABHAY KUMAR

Supercyclicity of C₀-Semigraoups.

Supervisor: Dr.Sachi Srivastava

Th 25360

Abstract (Not Verified)

Dynamics of linear operators is a topic that has had a rapid growth in recent years. In particular cyclicity, hypercyclicity, supercyclicity of bounded linear operators and hypercyclicity of strongly continuous operator semigroups have been extensively studied by several authors. In this thesis we undertake a systematic study of supercyclic strongly continuous operator semigroups. Thus far there have been only a few results in the existing literature regarding these. We give different supercyclicity criteria for \$C_0\$-semigroups and prove their equivalence. Several necessary conditions for supercyclicty of semigroups are given which are very helpful to rule out several classes of semigroups of operators from being supercyclic. We show that every bounded supercyclic \$C_{0}\$-semigroup is stable. Further we study solution semigroups of some PDEs and explore some variants of supercyclicity Finally, we study hypercyclicity and supercyclicity of unbounded operators and relate it to the Single Value Extension Property (SVEP) for unbounded operators. This allows us to prove that the sufficient conditions of Godefroy and Shapiro's criterion for hypercyclicity of \$C_0\$-semigroups are necessary as well.

Contents

1. Introduction 2. Supercyclic C_0 -semigroups and somewhere dense orbits 3. Supercyclocity criteria for C_0 -semigroups 4. Generation of supercyclic C_0 -semigroups 5. Supercyclic composition C_0 -Semigroups 6. Hypecyclicity and supercyclicity of unbounded operators. 7. References.

02. ANKIT KUMAR

Weyl type Theorems and other Spectral Properties of Bounded Linear Operators and Perturbations on Banach Spaces.

Supervisor: Dr. Anuradha Gupta

Th 25366

Abstract (Verified)

The thesis includes the study of spectral properties of bounded linear operators acting on a Banach space. We investigate common spectral properties of bounded linear operators A and B satisfying the operator equation AkBkAk = Ak+1 and BkAkBk = Bk+1 for some $k \in N$. We establish the Cline's formula for Drazin inverse in a ring with identity in the case when akbkak=ak+1 for some $k \in N$. We study the

Browder type theorems by means of generalized Drazin-Riesz invertible and generalized Drazin-meromorphic invertible operators. We obtain the necessary and sufficient conditions for a bounded linear operator T to be generalized Drazin-Riesz invertible or generalized Drazin-meromorphic invertible. Also, we obtain new characterizations of Browder type theorems by considering the equality between generalized Drazin-meromorphic invertible (generalized Drazin-meromorphic bounded below, generalized Drazin-meromorphic surjective, respectively) spectrum and generalized Drazin-meromorphic Weyl (generalized Drazin-meromorphic upper semi-Weyl, generalized Drazin-meromorphic lower semi-Weyl, respectively) spectrum. For bounded linear operators A and B, we generalize the Cline's formula for the case of generalized Drazin-meromorphic invertibility under the assumption that AkBkAk = Ak+1 for some $k \in N$. Further, we continue the study of Weyl type theorems. Property (UWII) is studied in connection with Weyl type theorems. We introduce properties (BR) and (BgR) for bounded linear operators. Also, the stability of these properties is examined under some commuting perturbations, for instance, nilpotent, quasi-nilpotent, finite-dimensional, compact, Riesz and algebraic commuting perturbations. We discuss the characteristics of quasi-Fredholm resolvent set. Also, we examine the stability of connectedness of quasi-Fredholm resolvent set under compact and weakly compact perturbations.

Contents

1. Introduction. 2. Common spectral properties and extension of cline's formula 3. Generalized drazin- meromorphic invertible operators and browder type theorems 4. Weyl type theorems and perturbations 5.Quasi-fredholm spectrum and compact perturbations. References.

03. CHAUHAN (Shipra)

Analysis of Robe's Finite Straight Segment Model Involving Perturbations: Oblateness, Coriolis and Centrifugal Forces.

Supervisor: Dr. Bhavneet Kaur

Th 25578

Abstract (Verified)

Robe in 1977 contemplated a new kind of restricted three-body problem, considering the more massive primary as a rigid spherical shell filled with a homogenous incompressible fluid, and the less massive primary is considered to be a point mass that lies outside the more massive primary. He analysed the motion of a small solid sphere that moves freely inside the more massive primary, under the in uence of buoyancy force of the fluid and attraction of the less massive primary. An equilibrium point at the centre of the more massive primary was found, and he discussed its linear stability for the circular and elliptical cases. His model proved out to be pivot point for the subsequent studies of finding equilibrium points and their linear stability by considering the various generalizations of this problem. The aim of this work is to study a more realistic model of the Robe's problem. The belief that celestial bodies as perfect spheres results in certain idealistic conditions. Most often, they are in irregular shapes. When considering irregularly shaped bodies, we can see that their key form attribute is body elongation. With this in mind, we conduct an analysis in which we approximated elongated body with a finite straight segment.

Contents

1. Introduction. 2.Existence and stability of the equilibrium points in the robe's finite straight segment model 3. Effect of perturbations in the coriolis and centrifugal forces in the robe's finite straight segment model 4. Perturbed robe's finite straight segment model with arbitrary density parameter 5. Robe's finite straight segment model under the effect of oblateness 6. Perturbed robe's finite straight segment model with oblatenesss. Bibliography.

04. DHARMENDRA KUMAR

Dynamical Structures of Solitary Wave Solutions for the Nonlinear Evolution Equations using lie Symmetry Analysis.

Supervisor : Dr. Sachin Kumar

Th 25352

Abstract (Verified)

In this thesis, we study the applications of Lie symmetry analysis to the nonlinear evolution equations which represent some of the important physical phenomena of the real world. Partial differential equations can be regarded as evolution equations on an infinite-dimensional state space. So, our primary objective in this word is to identify the Lie point symmetries of certain nonlinear partial differential equations in order to obtain solitary wave solutions. Furthermore, the dynamical structures of the obtained solution for the equations are discussed through their evolution profiles. The investigations carried out in this thesis are confined to the applications of symmetry analysis for five nonlinear partial differential equations which are the (2+1)-dimensional Pavlov equation, the (3+1)-dimensional extended Zakharov-Kuznetsov equation, the (3+1)-perturbed Zakharov-Kuznetsov equation, the (3+1)dimensional generalized shallow water wave equation and the (3+1)-dimensional generalized B-type Kadomstsev Petviashvili equation. The one-dimensional optimal system of Lie subalgebra is constructed by using Lie point symmetries through the construction of invariants and construction of the adjoint transformation matrix. The governing equations are reduced into a number of ordinary differential equations by applying similarity reductions. One of the reduced partial differential equations is dealt with a new generalized exponential rational function method, to obtain closed-form analytical solutions. Eventually, single soliton, quasi-periodic soliton, multisoliton, lump-type soliton, traveling wave, and solitary wave-interaction behavior are illustrated graphically through numerical simulation for physical affirmation of the results. Also, obtained solutions are the new solitary wave, multisoliton, and kink type which is significant in the field of applied mathematics.

Contents

1. Introduction 2. Invariant solutions and dynamics of solitons for (2+1)-dimensional Pavlov equation 3. Solitary wave solutions of (3+1)-dimensional extended zakharov-kuznetsov equation 4. Solitary wave solutions of (3+1)-perturbed zakharov-kuznetsov equation 5. Periodic solitary wave solutions of (3+1)-dimensional generalized shallow water wave equation 6. Invariant solutions of (3+1)-dimensional generalized B-type kadomstsev petviashvili equation 7. Concluding remarks and future scope. References.

05. GUPTA (Bhawna)

Hankel Operators on Some Spaces of Analytic Functions.

Supervisor: Dr.Anuradha Gupta

Th 25364

Abstract (Verified)

The thesis includes the study of Hankel operators and their several generalizations on various spaces of analytic functions like Hardy space over polydisk, Bergman space, Fock-Sobolev spaces, etc. In this direction, algebraic as well as spectral properties of kth-order (slant Toeplitz + slant Hankel) operators are explored on the Hardy space for essentially bounded measurable symbol functions. It is shown that if k = 2, then the Coburn type theorem holds for these operators. The notion of kthorder slant little Hankel operator is defined on the Bergman space and its point spectrum is evaluated for some special symbol function. It turns out that the spectrum and approximate point s Further, the weighted composition operators and k-Hankel composition operators are studied on weighted Hardy spaces and derivative Hardy space, respectively, and their operator-theoretic properties are obtained. The necessary and sufficient conditions are investigated for the difference operator of weighted composition operators to become compact operator. A compact subset of non-negative real numbers is evaluated which contains essential norm of weighted composition operator. The sufficient condition is investigated for k-Hankel composition operator to be Hilbert-Schmidt operator. Next, the notions of partial asymptotic Hankel operators, asymptotic Hankel operators and ith-partial Hankel operators are introduced on the Hardy space over polydisk. The characterization is obtained for the compactness of ith-partial Hankel operators. Properties of asymptotic Hankel operators and partial asymptotic Hankel operators are also explored for weak and strong convergence cases. It is shown that the Toeplitz algebra is contained in the set of all strong partial asymptotic Hankel operators. Moreover, Hankel operators on the Fock-Sobolev spaces are studied in terms of BMOrp and VMOrp spaces pectrum of this operator are same for the special symbol. Further, the weighted composition operators and k-Hankel composition operators are studied on weighted Hardy spaces and derivative Hardy space, respectively, and their operatortheoretic properties are obtained. The necessary and sufficient conditions are investigated for the difference operator of weighted composition operators to become compact operator. A compact subset of non-negative real numbers is evaluated which contains essential norm of weighted composition operator. The sufficient condition is investigated for k-Hankel composition operator to be Hilbert-Schmidt operator. Next, the notions of partial asymptotic Hankel operators, asymptotic Hankel operators and ith-partial Hankel operators are introduced on the Hardy space over polydisk. The characterization is obtained for the compactness of ithpartial Hankel operators. Properties of asymptotic Hankel operators and partial asymptotic Hankel operators are also explored for weak and strong convergence cases. It is shown that the Toeplitz algebra is contained in the set of all strong partial asymptotic Hankel operators. Moreover, Hankel operators on the Fock-Sobolev spaces are studied in terms of BMOrp and VMOrp spaces.

Contents

1. Introduction 2.(Slant Toeplitz +Slant Hankel) operators and their generalization 3. Slant little hankel operators 4. Composition and hankel composition operators 5. Hankel and asymptotic hankel operators 6. References.

06. JYOTI

Matrix-Valued Wave Packet Frames.

Supervisor: Prof. C. S. Lalitha

Th 25362

Abstract (Verified)

The thesis entitled 'Matrix-Valued Wave Packet Frames' is the study of frames of matrix-valued wave packet systems in matrix-valued function spaces. The thesis begins with the introduction of matrix-valued wave packet frames in matrix-valued function spaces. The interplay between matrix-valued wave packet frames and their associated atomic wave packet frames is discussed. The duality of matrix-valued wave packet frames in terms of their associated atomic dual frames is also discussed. A characterization of matrix-valued dual window functions in terms of orthogonality of wave packet Bessel sequences has been given. Some frame properties of finite sums of matrix-valued wave packet frames and an application of frame bounds associated with finite sums of frames to the frame algorithm have been obtained. The WH-packets of matrix-valued wave packet systems, which is a system of vectors analogous to Aldroubi's model for explicit expression of vectors, including frame vectors, in terms of a series associated with a given frame is studied. Different necessary and sufficient conditions for the existence of frames of WHpackets have been obtained. A Paley-Wiener type perturbation theorem for frames of WH-packets is also given. Finally, we study matrix-valued wave packet frames for subspaces under the name K-frames in matrix-valued signal spaces. A necessary and sufficient condition for the existence of K-matrix-valued wave packet frames under a suitable assumption on a trace functional associated with K is given. A characterization of K-matrix-valued wave packet frames in terms of atomic systems has been given. In the end, it is shown that K-matrix-valued wave packet frames are stable under small perturbations of the corresponding window functions.

Contents

1. Introduction 2.Frames of matrix-valued wave packet systems 3.Sums of matrix-valued wave packet frames 4. WH-packets of matrix-valued wave packet frames 5.K-matrix-valued wave packet frames 6. References.

07. Karuna

Semicontinuity of Solution Maps and External, Internal and Essential Stability in Set Optimization.

Supervisors: Prof. C. S. Lalitha

Th 25353

Abstract (NotVerified)

The purpose of the thesis is to study various aspects of stability in set optimization, in particular, continuity of solution set maps, convergence of solution sets and essential stability. The thesis comprises of six chapters. Chapter 1 is introductory and deals with some basic notions and concepts to be used in the subsequent chapters. We also provide an introduction to certain stability aspects in vector and set optimization. Chapter 2 deals with the study of continuity aspect of stability for parametric set optimization problems. The upper and the lower semicontinuity of solution set maps is established by using a notion of strict quasiconvexity. Chapter 3 deals with the study of convergence aspect of stability in set optimization by

perturbing the feasible set. In the given space, convergence of solution sets is established in Painlevé-Kuratowski sense, whereas in the image space, this aspect is studied in terms of external and internal stability in Hausdorff and Painlevé-Kuratowski sense. Further, we study this aspect in Chapter 4 by perturbing the objective function. We study the external and internal stability of solution sets in the framework of gamma convergence for set-valued maps. Further, using strict quasiconvexity notion, the results are deduced in the given space in Painlevé-Kuratowski sense. Chapter 5 deals with essential stability in set optimization in terms of essential sets of the solution sets. We characterize essential sets of efficient solution set in terms of an equivalent set corresponding to some efficient solution. We also study essential sets in the image space. In Chapter 6, we perturb both the feasible set and the objective function and study the convergence and continuity aspects of stability both in the given and the image space, by using a notion of topological convergence. We also investigate the convergence aspect using scalarization.

Contents

1.Introduction 2. Semicontinuity of solution set maps in parametric set optimization 3.External and internal stability via hausdorff and painleve- kuratowski convergence 4.External and internal stability via gamma convergence 5. Essential stability of solutions sets 6.Convergence and semicontinuity of minimal solutions using topological convergence on power sets. Bibliography.

08. KHARBANDA (Harsha)

Modeling and Qualitative Dynamics in Prey-Predator Interactions.

Supervisor: Dr. Sachin Kumar

Th 25350

Abstract (Verified)

The study of mathematical modeling is an important area of research. The present thesis addresses to the analysis of the various phases of the interactions of prey and predator population in the ecosystem. The dynamic relationships between species and their complex properties are at the heart of many ecological and biological processes. After formulating various mathematical models under assumptions, a variety of analytical methods and useful tools of ordinary differential equations and dynamical systems are utilized to analyze the proposed prey-predator models. Chapter 1 contains basic definitions and concepts required to prepare the background for study. In chapter 2, a prey-predator model in the presence of group defense and non-linear harvesting in prey is proposed and analyzed. In chapter 3, the complex dynamical behavior of a three dimensional model consisting of one prey and two predators involving Beddington- DeAngelis and Crowley-Martin functional responses is investigated. Chapter 4 is based on the dynamics of a stage-structured predator-prey system which incorporates cannibalism in the predator population and harvesting in both population. In chapter 5, a four dimensional ecoepidemiological system consisting of susceptible prey, infected prey, vaccinated prey and predator is analyzed. In these chapters, the following results have been proved: (i) the positivity and boundedness of the solutions, (ii) existence of equilibrium points, (iii) local and global asymptotic stability around various equilibria, (iv) possible bifurcations around different equilibria, (v) optimal harvesting policy using Pontryagin's maximum principle, (vi) sensitivity analysis, and (vii) chaos detection by examining Lyapunov exponents and Lyapunov dimension. The proposed models have been solved numerically via Mathematica and MATLAB. The thesis ends with conclusions and future direction in this area followed by a list of exhaustive references.

Contents

1. Introduction 2. Dynamics of a prey-predator model with group defense and non-linear harvesting 3. Stability analysis of one prey and two predators with two functional responses 4. Optimal control and existence of chaos in a cannibalistic prey-predator model with harvesting 5. An eco-epidemiological system with vaccination and migration in prey 6. Conclusions and possible future directions. Appendix. Reference.

09. KHAN (Abdul Gbdul)

Variants of Expansivity, Shadowing, Persistence and Walters Stability Theorem in Pointwise Dynamics.

Supervisor: Prof. Tarun Kumar Das

Th 25580

Abstract (Verified)

Thesis is divided into five chapters. In Chapter-1, we present the historical development of pointwise dynamics and discuss several existing useful dynamical properties. In Chapter-2, we define topologically transitive points, specification points, periodic specification points, minimally expansive points and \$GH\$-stable points. We prove that every continuous surjective map of a perfect metric space with pointwise periodic specification property is Devaney chaotic. Every uniformly continuous surjective map with at least two distinct specification points has positive entropy; the unit circle does not admit any expansive homeomorphism and every minimally expansive shadowable point is topologically stable and \$GH\$-stable. In Chapter-3, we define pointwise expansive measures, \$\mu\$-uniformly expansive points, \$\mu\$-shadowable points and strong \$\mu\$-topologically stable points. We prove that for every pair of points p and q, the set of all points which are positively asymptotic to p and negatively asymptotic to q, has measure zero with respect to any pointwise expansive measure. It is also shown that every \$\mu\$-uniformly expansive \$\mu\$-shadowable point is a strong \$\mu\$-topologically stable point. In Chapter-4, we define average shadowable measures, almost average shadowable measures, pointwise weakly topologically stable homeomorphisms and average persistent homeomorphisms. We prove that the set of all average shadowable measures is dense in the space of all Borel probability measures if and only if the set of all average shadowable points of map is dense in space. We also prove that every equicontinuous pointwise weakly topologically stable homeomorphism is persistent; every pointwise minimally expansive homeomorphism with persistent property is pointwise weakly topologically stable and every mean equicontinuous pointwise weakly topologically stable homeomorphism is average persistent. In Chapter-5, we obtain sufficient conditions under which the limit of a sequence of functions exhibits expansivity, \$\mu\$-uniform expansivity, shadowing, topological transitivity, topological stability and their variants at a point.

Contents

1. Introduction 2. Specification, uniformly and minimally expansive points 3. Expansive and shadowable points for measures 4. Average shadowing and persistent property 5. Dynamics under orbital convergence. Bibliography. List of Symbols. Index.

10. KHUSHBOO

Scalarization of Minimal Solutions in Vector and Set Optimization.

Supervisor: Prof. C. S. Lalitha

Th 25579

Abstract (Not Verified)

Scalarization is one of the classical tools employed to characterize minimal solutions of vector or set optimization problems in terms of minimal solutions of associated scalar problems. Most frequently used nonlinear scalarizing functions are Gerstewitz function, oriented distance function, and their generalizations. Another nonlinear scalar function unifying these functions is based on order representing and order preserving properties. This thesis aims to study various scalarization schemes in vector and set optimization using these nonlinear scalar functions. For vector optimization problems, we characterize minimal and approximate minimal solutions of a set with respect to a preference relation induced by an arbitrary set using scalar functions satisfying order representing and order preserving properties. We also characterize minimal solutions using oriented distance function which satisfies these properties. For set-valued optimization problems, we extend the notion of minimal solutions of vector optimization with respect to an arbitrary set in vector and set criterion approaches and establish links between both the solution concepts. We introduce a generalized Gerstewitz function and establish scalarization of minimal solutions in set criterion sense. For these minimal solutions, we provide an existence result and sufficient conditions for the closedness of the solution set. consider a new solution concept with respect to an arbitrary set and establish scalarizations using a generalized oriented distance function of sup-inf type. We also provide an existence result for minimal solutions and discuss stability via scalarization. Further, we define two generalizations of oriented distance function over family of sets. Using a partial set order relation we characterize minimal solutions using the first generalized oriented distance function and establish two existence results for minimal solutions. We scalarize three types of minimal solutions with respect to a quasi-set order relation using the second oriented distance function and study the lower semicontinuity of this function.

Contents

1. Introduction 2. Scalarization of minimal and approximate minimal solutions with respect to an arbitrary preference relation in vector optimization 3. Scalarization of minimal solutions with respect to an arbitrary preference relation in set optimization 4. Scalarization and convergence of a notion of minimal solutions with respect to an arbitrary preference relation in set optimization 5. Scalarization of minimal solutions with respect to a partial order relation in set optimization 6. Scalarization of minimal solutions with respect to a quasi order relation and continuity of associated scalar maps. Bibliography.

11. LALWANI (Kushal)

Topological Dynamics for Generalized Semigroups: Attractors, Recurrence and Escaping Set.

Supervisor: Dr. Sanjay Kumar

Th 25355

Abstract (Verified)

The present thesis considers the dynamical properties of a generalized semigroup of continuous self maps of a topological space. Considerations about limiting behaviour and corresponding extension of results become of prime importance. The principal aim is to see, to what extent, the theory of classical dynamics can be applied in this more general setting. We have considered the sequential approach to study the limiting behaviour of iterates. In Chapter 1, we review and collect some basic concepts and results from the classical theory of topological dynamics, which will be used throughout the thesis. Also, we introduce the notion of generalized semigroup here. In Chapter 2, we have developed a parallel theory of recurrence in the context of a generalized semigroup analogous to its counterpart in the classical theory. In Chapter 3, we define the concepts of chain recurrence, topological transitivity and chain transitivity, and subsequently proceed the theory. The definition of attractor in the context of generalized semigroup is incorporated from a detailed study of the concept. Chapter 4 is devoted to the dynamics on noncompact topological spaces. Most of the non-vacuous results in dynamical systems depends on the convergence of the sequence of iterates which is easily ensured by the compactness of the space. Here, we study the eventual behaviour of the sequence of iterates of a point when the space is not compact. Lastly, we have revisited characterization of chain recurrent set in terms of corresponding attractor and repeller pairs in noncompact metric space.

Contents

1.Introduction 2. Recurrence Properties 3. Chain recurrence and attractors on compact metric spaces 4. Dynamics on noncompact spaces 5. References. Index.

12. PANDEY (Shesh Kumar)

Multivariate Analogue of Slant Toeplitz Operators and Their Compression.

Supervisor: Dr. Gopal Datt

Th 25359

Abstract (Verified)

The ultimate goal of the thesis is to analyze the multivariate version of slant Toeplitz operators, introduced by M.C. Ho, on the Lebesgue space L2(Tn) and their compression to the Hardy space H2(Tn) . It deals with several algebraic and structural properties along with spectral properties of concerned operators on the Lebesgue and Hardy spaces of n- torus. In the thesis, we introduce the kth- order slant Toeplitz operator on the Lebesgue space L2(Tn) for a fixed integer k≥2, which is characterized by a system of operator equations, Mz1j1z2j2...znjn A = AMz1kj1z2kj2...znkjn for (j1, j2, ..., jn) ∈Bn , where Bn is the standard basis of Rn. Next, we solve the system Mz1j1z2j2...znjn A = $\lambda(j1+j2+$ AMz1kj1z2kj2...znkjn for (j1, j2, ..., jn) \in Zn and $\lambda \in C$, and utilize its findings to generalize the notion of slant Toeplitz operators defined on L2(Tn), which are named as kth- order λ -slant Toeplitz operators. In addition, we discuss the notion of essentially kth- order λ-slant Toeplitz operator on L2(Tn) with the aid of Calkin algebra. The compression of kth- order slant Toeplitz operators and kth- order λ slant Toeplitz operators to the Hardy space H2(Tn) are also discussed. Characterization is obtained for an operator V on H2(Tn) to be compression of a kthorder slant Toeplitz operator (kth- order λ -slant Toeplitz operator). Further, we discuss essential compression of kth- order λ-slant Toeplitz operators. Finally, we study multi-indexed multivariate version of kth- order slant Toeplitz operators, where we obtain a characterization for kth- order slant Toeplitz operators on the Lebesgue space L2(Tn), where k=(k1, k2,...,kn), $ki \ge 2$ for all i.

Contents

1. Introduction 2.Multivariate version of slant toeplitz operators 3. Multivariate version of generalized slant toeplitz operators 4. Compression of multivariate version of slant toeplitz operators 5. Multi-indexed multivariate slant toeplitz operators. References.

13. POONAM RANI

Normality for Families of Meromorphic Functions.

Supervisors: Dr. Sanjay Kumar

Th 25363

Abstract (Verified)

This thesis deals with the study of notion of normal family in complex analysis. The theory of normal family was introduced by Montel in 1907. This beautiful theory was developed through years with the Nevanlinna theory and Ahlfors's theory of covering surfaces. Exploiting the work of Montel, Fatou and Julia laid the foundation of modern theory of complex dynamics. This shows the importance of normal families in function theory. In 1975, Zalcman proved a renormalization result, now known as Zalcman's lemma. Later Schwick in 1992, discovered a connection between normality and shared values. By using results of value distribution theory and Zalcman's lemma we proved several normality criteria for families of meromorphic functions via the techniques of shared functions in plain domains.

Contents

1.Introduction 2. Sharing of Holomorphic functions and normality 3. Normality for differential polynomial concerning shared functions 4. Normality criteria concerning the zeros of differential polynomial. References.

14. Prajapati (Tarachand)

C₀ Senugriyos of Operators on Non-Commutative Lp-Spaces.

Supervisor : Dr. Sachi Srivastava and Dr. Ratikanta Panda Th 25357

Abstract (Not Verified)

This thesis studies properties of C0-semigroups on non-commutative Lp-spaces. The C0-semigroup is the natural mathematical model for Cauchy problems and the properties of the solution of the Cauchy problem are obtained by knowing the behaviour of the C0-semigroup. For applications, it is useful to describe the properties of a semigroup in terms of its generator, as this gives valuable information about the solutions of the Cauchy problem even though the solutions may not be known explicitly. While the behaviour of C0-semigroups defined on classical Lp-spaces has been extensively studied and is well understood, the non-commutative case has remained largely unexplored so far. We examine the results and properties of C0-semigroups that are valid for classical Lp-spaces and obtain non-commutative versions for these. The growth bound of a positive C0-semigroup on classical Lp-space coincides with the spectral bound of its generator. Analogs of this

equality, often called the Lyapunov property for C0-semigroup on non-commutative Lp-spaces are obtained. We also study to what extent eventually norm continuous semigroups on non-commutative Lp-spaces can be characterized in terms of the decay of the resolvent of the generator. Such a characterization holds for positive C0-semigroups on classical Lp-spaces, but we see that in the non-commutative setting some extra technical conditions are needed. We also identify some properties of C0-semigroups that extrapolate on non-commutative Lp scale. These include holomorphy of the C0-semigroup, boundedness of the generator, eventual norm continuity, compactness of the resolvent of the generator and stability of the C0-semigroup. These extrapolation results give us for free the same properties on Lp-spaces which are more difficult to study individually. We focus, in particular, on the problem of the invariance of the spectrum of the generator.

Contents

1. Introduction 2. C_0 -Semigroups 3. Lyapunov property for semigroups on Lp(M,T) 4. Extrapolation properties of C_0 -Semigroups 5. Eventual norm continuity. 6. Bibliography, Index.

15. RAHUL KUMAR

Ring Extensions with Various Conditions on Intermediate Rings.

Supervisors: Dr. Atul Gaur

Th 25358

Abstract (Not Verified)

The thesis is apportioned into seven chapters. A brief organization of the thesis is as follows: Chapter-1 entitled Introduction gives a brief background for motivating the present study along with basic concepts and definitions from ring theory which is relevant to the contents of the thesis. Chapter-1 entitled Introduction gives a brief background for motivating the present study along with basic concepts and definitions from ring theory which is relevant to the contents of the thesis. Chapter-2 entitled Maximal subrings of a ring study rings which do admit maximal subrings and study minimal ring extensions of some known rings. Chapter-3 entitled λextension of rings is devoted to study λ -extensions more deeply and extend the concept of λ -domain. Chapter-4 entitled Δ -extension of rings deals with the study of the Δ -extension more deeply and extend the concept of Δ -domain. In Chapter-5 entitled Maximal non λ -subrings, we introduce the concept of maximal non λ subrings of a ring and study its properties and characterizations. Chapter-6 entitled Generalization of maximal non valuation subrings generalizes the concept of maximal non valuation subrings of a domain. Chapter-7 entitled Maximal non φ-chained rings and maximal non chained rings gives an interplay between maximal non P-subrings and ϕ -chained rings.

Contents

1. Introduction 2. Maximal subrings of a ring 3. –Extension of rings 4. ^-Extension of rings 5. Maximal non^-subrings 6. Generalization of maximal non valuation subrings 7. Maximal non-chained rings and maximal non chained rings. References.

16. RAM RATAN

Cryptanalysis of Cipher Systems using Data Characteristics, Attacks and Fuzzy Computational Methods.

Supervisors : Dr. Arvind

Th 25356

Abstract (Verified)

With the advancement in digital technologies and information systems, the use of visual data has emerged. Security of vital visual data is one of the important requirement to safeguard information from adversaries. Security of data can be achieved by a cipher system using cryptography which uses an encryption algorithm to encrypt plain data, and a decryption algorithm to decrypt encrypted data. Symmetric key cryptography is being used widely to achieve the security of data. For such cipher systems, the use of chaotic, stream, and selective encryption schemes are reported. Many encryption schemes perform pixel permutation, substitution or in combination of both. Some of the encryption schemes claimed secure had been found insecure as the key and data can be obtained. Hence, prior to use of any cipher system, it should be analysed for cryptanalytic attacks to assure its security strength. Also, one requires a decision methodology to select best suitable cipher system. The thesis studies cipher systems with respect to cryptanalytic attacks, countermeasures, and select the best cipher system based on security attributes to meet customer requirements. It studies the bit-plane image characteristics of images and their applications in cryptanalysis of encryption schemes. It carries out the cryptanalysis of image encryption schemes using neighbourhood similarities and divide-and-conquer attack, and two-messages-on-same-key attack for extracting meaningful information from encrypted images. Randomness tests are presented to study two-dimensional bit-plane level characteristics of cipher key sequences. A fuzzy soft set based decision methodology is presented for selecting a best cipher system. Cryptanalysis provides intelligible decrypted images, new randomness tests trace the non-randomness not traceable by other tests, and a fuzzy decision selects a best one cipher system successfully. Cryptanalysis methodologies reported in the thesis are very useful and can be applied in solving future problems of pattern recognition, data analysis, and cryptanalysis of ciphers.

Contents

1. Introduction 2.Cryptanalysis of ciphers using bit-plane measures 3. Cryptanalysis of ciphers using divide-and-conquer attack 4. Cryptanalysis of ciphers using two message-on-same-key attack 5. Cryptanalysis of ciphers using bit-plane randomness tests 6. Cryptanalysis for best cipher system selection using fuzzy methodology 6. Conclusions and Research Directions. References.

17. ROHILLA (Manu)

Fixed Point Theorems for Contractive Mappings in Generalized Metric Spaces.

Supervisor : Dr. Anuradha Gupta

Th 25351

Abstract (Verified)

The thesis includes the study of fixed point theorems for contractive mappings in various generalizations of metric spaces. We generalize the notion of s-simulation function using c-class functions satisfying property cf. We introduce the notions of cf-s-simulation function and c-simulation function. We establish the existence and uniqueness of point of coincidence of a pair of self mappings in the setting of b-metric spaces via cf-s-simulation functions. We obtain common fixed point results of self mappings in the context of complete complex valued metric spaces via c-simulation functions. We introduce the notion of quasi-partial b-metric-like spaces as a generalization of quasi-partial b-metric spaces. We obtain the existence and

uniqueness of fixed points of (α, ψ) g-contractive mappings on quasi-partial b-metric-like spaces endowed with a graph. Also, the influence of errors on the convergence of infinite products of weak quasi-contraction mappings in b-metric spaces is explored. We obtain convergence results under the assumption that the exact infinite orbits converge and the sequence of computational errors converges to zero. Also, we have formulated weak ergodic theorems in b-metric spaces. Further, we introduce the notion of p-cyclic contraction and p-cyclic nonexpansive mappings. We formulate necessary conditions which ensure the existence of a coupled best proximity point of p-cyclic contraction and p-cyclic nonexpansive mappings in the setting of uniformly convex banach spaces. Also, we investigate ulam-hyers stability of the coupled best proximity point problem in the case of p-cyclic contraction and p-cyclic nonexpansive mappings.

Contents

1. Introduction 2. Fixed point theorems via simulation functions 3. Fixed point theorems of G-Contractive mappings in quasi-partial b-metric-like spaces 4. Inexact infinite products of weak quasi-contraction mappings in b-Metric spaces 5. Coupled best proximity point and ulam- hyers stability.

18. SACHAN (Prachi)

Basins of Convergence in Axisymmetric Restricted Five-Body Problem.

Supervisors: Prof. Rajiv Aggarwal

Th 25354

Abstract (Not Verified)

Undoubtedly, the study of the motion of the test particle moving under the gravitational influence of N -body is one of the well intriguing problem. A plethora of research papers are available where the N -body problem is discussed in details under various perturbations. The present thesis is composed of the four problems where we have analysed the effect of various perturbations on the topology of the basins of convergence by deploying multivariate version of the Newton-Raphson iterative scheme in the restricted five-body problem. In any dynamical system the knowledge of the basins of convergence linked with the equilibrium points is of paramount importance, since these basins reflect some of the most essential properties of the dynamical system. We have considered mainly two type of configurations in the restricted five-body problem. First, the axisymmetric restricted five-body problem: the convex & concave configuration, and second, the photogravitational version of the restricted five-body problem of Ollöngren (1988). The chapter wise description of the thesis is as follows: Chapter 1 is introductory in nature and we have revealed the literatures related to N- body problem and methodology which will be used to investigate the existence and stability of the dynamical system, and the regions of possible motion. Moreover, the multivariate version of the Newton-Raphson iterative scheme is revealed to solve the system of non-linear equations and to deploy the same to unveil the topology of the basins of convergence associated to the equilibrium points. Additionally, we have revealed the development of the central configuration of four-body problem by Erdi and Czirják (2016), in which they have discussed the convex and concave configurations of its. Chapter 2 deals with the topology of the basins of convergence in the axisymmetric restricted five-body problem with convex configuration. We have shown that how the existence of total number of libration points and their positions depend on the angle parameters α and β . Further, stability of the libration points is discussed in linear sense. Additionally, the evolution of the structure of the three dimensional zero velocity surfaces for several values of the Jacobi constant C and for the angle parameters α and β are drawn. The multivariate version of the Newton-Raphson iterative scheme is used to unveil the domain of the basins of convergence associated with the libration points on two-dimensional configuration planes. In addition, we have presented that how the basins of convergence with corresponding number of required iterations are correlated. In Chapter 3, we have presented the study of the topology of basins of convergence in the axisymmetric restricted five-body problem with concave configuration in two different cases. We have used the same methodology developed in Chapter 2 to discuss the parametric evolution of the equilibrium points and their stability. Moreover, the study of the basins of convergence by deploying the multivariate version of the Newton-Raphson iterative scheme unveil the fact that the angle parameters lpha and etahave substantial effect on their topology. In addition, the corresponding probability distributions of the required iterations are also unveiled. Chapter 4 deals with the axisymmetric restricted five-body problem with the effect of small perturbations in the Coriolis and centrifugal forces. We have shown how the effect of the small perturbations in the Coriolis and centrifugal forces influence the existence, locations of the libration points and their stability. The stability of the libration points for various combinations of the angle parameters is highly effected by the small perturbations in both the forces i.e., the Coriolis and centrifugal forces. We revealed that how the topology of the zero velocity surfaces, and of course the linked zero velocity curves change as a function of the parameter ϕ due to effect in the centrifugal force. It has been shown that the regions of motion are affected by the centrifugal force parameter but is independent of the Coriolis force parameter. In Chapter 5, we have studied the photogravitational restricted five-body problem in the Ollöngren (1988) with the effect of the Coriolis and centrifugal forces. In this chapter, the existence and stability of the in-plane equilibrium points and the possible regions of motion are explored and analysed numerically, under the combined effect of small perturbations in the Coriolis and centrifugal forces when the primaries are source of radiation. Moreover, the multivariate version of the Newton-Raphson iterative scheme is applied in an attempt to unveil the topology of the basins of convergence linked with the libration points as function of radiation parameters, and the parameters corresponding to the Coriolis and centrifugal forces.

Contents

1. Introduction 2. The newton-raphson basins of convergence of the libration points in the axisymmetric restricted five-body problem: the convex configuration 3.On the newton-raphson basins of convergence associated with the libration points in the axisymmetric restricted five-body problem: the convex configuration 4. The effect of small perturbations in the coriolis and centrifugal forces in the axisymmetric restricted five-body problem 5. On the perturbed photogravitational axisymmetric restricted five-body problem: the analysis of newton-raphson basins of convergence. Bibliography.

19. SHAH (Sarswati)

Evolution of Nonlinear waves in Quasilinear Hyperbolic Systems.

Supervisor: Dr. Randheer Singh

Th 25361

Abstract (Not Verified)

Hyperbolicity and nonlinear wave are vital characteristics of aid dynamics, aerodynam- ics, nuclear sciences and of various problems that arise in mathematical physics. The the-ory of quasilinear hyperbolic partial di erential equations (PDE's) gives the mathematical demonstration of shock waves. Many real-world phenomena arising in mathematical physics, continuum mechanics, represented by quasilinear hyperbolic PDE's whose solutions are non-linear waves involving shocks. The knowledge of nonlinear waves is vital in aerodynamics for spacecraft propulsion, in medical sciences to disintegrate kidney stone, in black hole theory etcetera. Over a hundred years ago, the study of nonlinear waves got much attention with the pioneering work of Stokes [1], Earns haw [2], Riemann [6] and Hadamard [7] and it con-tinued at an advancing pace, with appreciable development in recent years. The early work in this area is described by Courant and Friedrichs [4], Boillat et. al [5] and Sharma [3]. In the thesis, we have considered the quasilinear hyperbolic systems describing planarand radially symmetric ow of reacting polytropic gases, real dusty reacting gases and non-ideal relaxing gases. We have investigated the certain aspects of nonlinear waves such as singular surface, weak shock, characteristic shock along with formation of the shocks from initial data and their interactions. The thesis consists of six chapters. The descriptions of the chapters are brie y as per the following. The central focus was to know how children and adolescents construct their sense of morality and to locate these constructions and process of moral development, in the dimensions of age, socio economic status, gender and religion. The study was conducted in two phases. Phase 1 had 120 participants; tool consisted of eight moral dilemmas with multiple questions. Phase 2 had 48 participants; tool was a thematic interview schedule. Participants were adolescents (first year undergraduate students of various colleges of Delhi). There was an equal representation of gender (boys and girls), socio-economic background (lower, middle, and high), and religion (Hindu, Muslim, Sikh, and Christian).

Contents

1. Introduction 2. Evolution of singular surface and interaction with a strong shock in reacting polytropic gases using lie group theory 3. Imploding shocks in real reacting gases with dust particles 4. Collision of a steepened wave with a blast wave in dusty real reacting gases 5. Propagation of non-planar weak and strong shocks in a non-ideal relaxing gas 6. Interaction of a characteristic shock and a singular surface in a non-ideal reacting gas with dust particles. Bibliography.

20. SUBODH KUMAR

Codes Dealing with Repeated Burst and Key Error Control with Weight Distribution. Supervisors: Dr. Pankaj Kumar Das and Anupama Panigrahi Th 25365

Abstract (Not Verified)

The whole coding theory is developed to find the solution to rectify the errors from the corrupted messages with better efficiency and simple encoding and decoding methods. This thesis deals with mainly detection, location, correction and weight distributions (Hamming and Homogenous) of burst, repeated burst and key errors. For the results, we mostly follow combinatorial approach. The contents of this thesis are arranged into six chapters. The first chapter is the Introduction. In this chapter, we present a brief history of the subject and then preliminary works and results required for the study that comes in the subsequent chapters. Error-locating codes (EL codes) are very important for an application to memory systems that are constructed from byte-organized memory chips. In the second chapter, we consider this situation and obtain necessary and sufficient conditions for the existence of EL-codes capable of locating burst/repeated burst errors whether it is confined to one sub-block or spread over to the adjacent sub-block.

Contents

1. Introduction 2. Location of burst and repeated burst errors in single and adjacent sub-blocks 3. Codes detecting, locating and correcting of repeated burst errors prevailing in multiple sub-blocks 4. Linear codes over Z_{q1} correcting burst and repeated burst errors with homogeneous weight 5. Correction of low density key errors and weight distribution of key errors 6. Location and correction of key errors blockwise. Bibliography.

21. SUMIT

Variants of Hurewicz and Menger Covering Properties in Topological Spaces.

Supervisor: Prof. B. K. Tyagi

Th 25369

Abstract (Verified)

Chapter 1, contains historical developments and basic concepts and notations of Hurewicz and Menger spaces. We have tried to include enough basic concepts here to make the thesis self contained. In Chapter 2, we studied some covering properties in topological spaces defined by preopen sets. We introduced the pre Menger property, the almost pre Menger property and their star versions and investigated the relationships between them. We also studied the topological properties of the preMenger property, the almost preMenger property and their star versions. In Chapter 3, we proved that the set-Menger, set-Hurewicz and set-Rothberger properties, introduced in [15], are actually another view of the Menger, Hurewicz and Rothberger properties respectively, while the weakly set-Menger and weakly set-Rothberger properties are another view of quasi-Menger and quasi-Rothberger properties [19], respectively. We also proved that the almost set-Menger property in [15] is dielerent from the quasi-Menger property. Hence the [[15], Problem 4.20] is addressed. Further, we give the partial answer to Question 13 and Question 16 of Pansera [21]. It is shown that in a paracompact Hausdord space, almost star-Menger (weakly star-Menger) is equivalent to almost Menger (resp. weakly Menger). In Chapter 4, we continued the study of the ideal analogues of several variations of the Hurewicz property introduced by Das et al. [3, 8, 7] for example, the I-Hurewicz (IH), the star I-Hurewicz (SIH), the weakly I-Hurewicz (WIH) and the weakly star-I-Hurewicz (WSIH). We introduced the ideal analogues of some new variants of the Hurewicz property called the mildly I-Hurewicz and the star K-I-Hurewicz properties and explored their relationships with other variations of the I-Hurewicz property. We also studied the preservation of these properties under certain mappings. It is shown that several implications in the relationship diagram of these properties are reversible under certain conditions. For instance; (1) If a paracompact Hausdor space has the WSIH property then it has the WIH property. (2) If the complement of a dense set has the IH property then the WIH property implies the IH property and (3) If the complement of a dense set has the SIH property then the WSIH property implies the SIH property. In Chapter 5, we introduced the

ideal version of the star-C-Hurewicz property called star-C-I-Hurewicz (SCIH) property. We investigate the relationships among the SCIH and related properties. We study the topological properties of the SCIH property. This chapter generalizes several results of [25, 26] to the larger class of spaces having the SCIH properties. The star-C-I-Hurewicz game is introduced. It is shown that, in a paracompact Hausdor- space X, if TWO has a winning strategy in the SCIH game on X, then TWO has a winning strategy in the I-Hurewicz game on X. In Chapter 6, we introduced the ideal version of the absolutely strongly star- Hurewicz property called the ASSIH property. We investigated the relationships between the ASSIH and related properties and studied the topological properties of the ASSIH. This paper generalizes several results of [27, 28] to the larger class of spaces having the ASSIH properties.

Contents

1. Introduction 2. The preMenger covering properties 3. Variants of the menger covering property 4. Variants of the I-Hurewicz covering property 5. The star-C-I hurewicz property 6. The absolutely strongly star-I-hurewicz property. Conclusion. References.

22. TALWAR (Bharat)

Closed Lie Ideals of Certain Tensor Products of C*-Algebras and of Generalized group Algebras.

Supervisor: Dr. Ranjana Jain

Th 25367

Abstract (Verified)

We characterize the closed Lie ideals of certain tensor products of C*-algebras. While closed Lie ideals of operator algebras has been studied by many mathematicians, not much was known about the closed Lie ideals of tensor products. Can closed Lie ideals of tensor products be determined with the help of closed Lie ideals or closed ideals of individual Banach algebras? Even though the closure of the tensor product of two closed Lie ideals may not give a closed Lie ideal, some positive results can be obtained. In certain cases, a complete list of closed Lie ideals of tensor products is also given. The closed Lie ideals of generalized group algebras (roughly speaking, Avalued integrable functions on the Borel sigma-algebra of a locally compact group, where A is a Banach algebra) are also characterized. This characterization is given in terms of the elements and modular function of the group and elements of A. We than find the conditions under which closed Lie ideals of the Banach algebra A gives rise to closed Lie ideals of generalized group algebras. Even though it has been around five decades since we know that the Banach space projective tensor product of two Banach algebras is itself a Banach algebra, it is still not known whether center of this tensor product is isomorphic to the tensor product of centers of the Banach algebras we started with. This statement is known to be true in the algebraic setting as well as if we start with C*-algebras. We have made some progress in this direction and gave an affirmative answer if we take one of the Banach algebras to be convolution algebra of a specific class of groups, the small invariant neighbourhood groups, and if we assume that the other Banach algebra is unital.

Contents

1. Introduction 2.Closed lie ideals of certain tensor products of C^* -algebras 3. Closed lie ideals of spatial tensor product of C^* -algebras 4. Lie ideals of generalized group algebras 5. Center of generalized group algebras. Bibliography. Index.

23. TARUN LATA

Byte Correcting linear Codes: Restricted Optimal Codes and Restricted Optimal Codes.

Supervisor: Dr.Rashmi Verma

Th 25368

Abstract (Not Verified)

The main objective of the present thesis is to discuss linear optimal codes and perfect codes in byte oriented communication channels/ memory systems. In this thesis we propose two new kind of burst errors termed as restricted burst error and solid restricted burst error which are the modification of the definition of burst and solid burst given by Fire (1959) and Schillinger (1964) respectively. Lower and upper bounds over the number of parity-check digits required for byte oriented linear codes that are capable of correcting restricted bursts as well as solid restricted bursts have been obtained. The thesis comprises of five chapters. Chapter 1 is introductory in nature. Chapter 2 deals with linear codes capable of correcting restricted bursts of length 2 or less. A parity-check matrix of a linear optimal code that can correct such bursts has been constructed by using the construction method given by Etzion (2001). Lower and upper bounds on the number of parity-check digits required for a linear code that can correct such errors within all the bytes of same size β have been obtained. Chapter 3 presents the general case of linear codes that are capable of correcting restricted bursts of length bi or less within the i-th byte of size β i. Different types of restricted bi-byte correcting non-binary linear optimal codes have been illustrated with examples. Solid restricted bi-byte correcting non-binary linear optimal codes have been studied in chapter 4. A construction method of a paritycheck matrix of such a code has been discussed. bi-byte correcting linear perfect codes have been discussed in chapter 5. Illustrations of different types of bi-byte correcting linear perfect codes have been presented.

Contents

1. Introduction 2. Restricted 2-burst correcting non-binary linear optimal codes 3. Restricted bi-byte correcting non-binary linear optimal codes 4. Solid restricted bi-byte correcting non-binary linear optimal codes 5. Bi-byte correcting linear perfect codes. References.

24. THAKUR (Rahul)

Variants of Sensitivity and Chaos on Semiflows

Supervisor: Prof. Ruchi Das

Th 25558

Abstract (Verified)

The research work done in the present thesis which is written in the area of topological dynamical systems, is devoted to the study of variants of sensitivity and chaos on semiflows. Several other chaos-related properties such as topological transitivity, mixing and denseness of periodic points are also studied. Moreover, several interesting examples or counterexamples are provided in most of the cases related to the results established in the thesis. The organisation of the thesis is as follows: Chapter 1 gives a historical background of the research problems studied and preliminaries required for the remaining chapters. Chapter 2 studies the notions of F-sensitivity, multi-F-sensitivity, (F1, F2)-sensitivity, F-transitivity and F-mixing,

where F, F1, F2 are Furstenberg families, for iterated function systems. We prove several results for these variants of sensitivity and transitivity. Chapter 3 introduces the concepts of multi-sensitivity with respect to a vector, N-sensitivity and strong multi-sensitivity for general semiflows. We also study these notions on product as well as hyperspatial semiflows. Moreover, we introduce and study several stronger forms of n-sensitivity such as multi-n-sensitivity, strong n-sensitivity, syndetic nthick n-sensitivity and thickly syndetic n-sensitivity semiflows. Chapter 4 studies how various types of chaos such as Devaney chaos, Poincare chaos, strongly Ruelle-Takens chaos and strongly Auslander-Yorke chaos carry over to finite as well as to countably infinite products of semiflows and vice versa. We also study some chaos-related notions such as topological transitivity, sensitivity and denseness of periodic points on product semiflows. Chapter 5 deals with the study of stronger forms of sensitivity such as multi-sensitivity, ergodic sensitivity, syndetic sensitivity and strong sensitivity on arbitrary products of semiflows defined on Hausdorff uniform spaces. We also obtain interrelations between various stronger forms of sensitivity of a semiflow and its induced hyperspatial semiflow. Moreover, Devaney chaos and Poincare chaos are also studied.

Contents

1. Introduction 2. Sensitivity and transitivity of iterated function systems via Furstenberg families 3. Variants of multi-sensitivity and n-sensitivity on semiflows 4. Variants of chaos on product semiflows 5. Stronger forms of sensitivity and chaos on arbitrary products of semiflows and on hyperspational semiflows. Bibliography. Index.

25. VIBHA

Starlikeness and Convexity of Certain Univalent and Entire Functions. Supervisors: Dr.Ajay Kumar, Dr. V. Ravichandran and Dr. Ratikanta Panda <u>Th 25349</u>

Abstract (Verified)

For the univalent functions f on the unit disk satisfying f(0)=0=f'(0)-1, sufficient conditions are obtained so that the function f satisfies $|(zf(z)/f(z))^2-1|<1$ and such a function is called a lemniscate starlike function. The technique of first and secondorder differential subordination has been used. The admissibility conditions are derived for lemniscate of Bernoulli and employed in order to prove the main results. Using the results so obtained, the geometric properties of certain entire functions have been discussed. Conditions on the associated parameters have been derived so that some normalizations of the generalized and "normalized" Bessel function, Lommel function of first kind, and the confluent hypergeometric function satisfy the inequality $|(1+zf'(z)/f(z))^2-1|<1$ and $|(f'(z))^2-1|<1$. Further, a function f is Janowski starlike if zf'(z)/f(z) lies inside the disk whose diametric endpoints are (1-A)/(1-B) and (1+A)/(1+B) for $-1 \le B < A \le 1$. The radii of lemniscate starlikeness and Janowski starlikeness have been determined for the normalizations of q-Bessel functions, Bessel function of first kind, and Lommel function of first kind. Corresponding radii for convexity have also been obtained for these functions. Besides, estimates are obtained for the initial coefficients of such functions for which the function and analytic extension of its inverse to the unit disk both belong to certain subclasses of univalent functions. The thesis also improves several existing results.

Contents

1. Introduction 2. Lemniscate starlikeness of univalent functions 3. Lemniscate convexity of certain special functions 4. Radius properties of some entire functions 5. Bounds for initial coefficients of Bi-univalent functions. References. Index.